Section 4.4
Exponential and Logarithmic Equations

Under the Sea !

Though it can be pitch black in the depths of the ocean, sunlight is visible as you get closer to the surface. About 12% of the surface sunlight reaches a depth of 20 feet and about 1.6% reaches to a depth of 100 feet.

In the applications of this section, you will use an exponential function to determine the depths that correspond to various percentages of light.

Objective #1: Use like bases to solve exponential equations.

Solved Problem #1

1a. Solve: $5^{3x-6}=125$.

$$5^{3x-6}=125$$
$$5^{3x-6}=5^3$$
$$3x-6=3$$
$$3x=9$$
$$x=3$$

The solution set is $\{3\}$.

1b. Solve: $8^{x+2}=4^{x-3}$.

$$8^{x+2}=4^{x-3}$$
$$(2^3)^{x+2}=(2^2)^{x-3}$$
$$2^{3(x+2)}=2^{2(x-3)}$$
$$3(x+2)=2(x-3)$$
$$3x+6=2x-6$$
$$x+6=-6$$
$$x=-12$$

The solution set is $\{-12\}$.

remember

$(x^m)^n = x^{m \cdot n}$

Pencil Problem #1

Look at your Card

1a. Solve: $4^{2x-1}=64$.

$4^{\boxed{2x-1}} = 4^{\boxed{3}}$

So $4^{\text{this}} = 4^{\text{that}}$ you know this = that

$2x-1=3$
$+1 \quad +1$
$2x=4$

$\boxed{x=2}$

1b. Solve: $8^{x+3}=16^{x-1}$.

$(2^3)^{x+3} = (2^4)^{x-1}$

$2^{3(x+3)} = 2^{4(x-1)}$

$2^{\boxed{3x+9}} = 2^{\boxed{4x-4}}$

$2^{\boxed{\text{this}}} = 2^{\boxed{\text{that}}}$ So $\boxed{\text{this}} = \boxed{\text{that}}$

$3x+9 = 4x-4$
$-3x+4 \quad -3x+4$
$\boxed{13 = x}$

$2^2=4$, $2^3=8$, $2^4=16$, $2^5=32$, $2^6=64$

$3^2=9$, $3^3=27$, $3^4=81$

$4^2=16$, $4^3=64$, $4^4=256$

$5^2=25$, $5^3=125$, $5^4=625$

$6^2=36$, $6^3=216$

$7^2=49$, $7^3=343$

$8^2=64$, $8^3=512$

$9^2=81$, $9^3=729$

$10^2=100$, $10^3=1000$

Remember $x \cdot \log_b m = \log_b m^x$

Both ways $\log_b m^x = x \cdot \log_b m$

Objective #2: Use logarithms to solve exponential equations.

✔ Solved Problem #2

2a. Solve: $10^x = 8000$.

Take the common log of both sides of the equation.

$$10^x = 8000$$
$$\log 10^x = \log 8000$$
$$x = \log 8000$$
$$x \approx 3.90$$

The solution set is $\{\log 8000 \approx 3.90\}$.

2b. Solve: $7e^{2x} - 5 = 58$.

Isolate the exponential expression, then take the natural log of both sides of the equation.

$$7e^{2x} - 5 = 58$$
$$7e^{2x} = 63$$
$$e^{2x} = 9$$
$$\ln e^{2x} = \ln 9$$
$$2x = \ln 9$$
$$x = \frac{\ln 9}{2}$$
$$x = \frac{\ln 3^2}{2}$$
$$x = \frac{2\ln 3}{2}$$
$$x = \ln 3$$
$$x \approx 1.10$$

See they added 5 then divided by 7 First

The solution set is $\{\ln 3 \approx 1.10\}$.

Pencil Problem #2

2a. Solve: $10^x = 3.91$.

Now when variable is a exponent and you cannot use your card you will need to follow these steps

1. get base an exponent by itself
2. take ln or log of both sides
3. use prop above to bring exponent out front
4. undo what is happening

① $10^x = 3.91$

② $\log 10^x = \log 3.91$

③ $x \cdot \log 10 = \log 3.91$ ($\log 10$ is $=1$)

④ $x = \log 3.91$

$\boxed{x = .592}$

2b. Solve: $7^{x+2} = 410$.

① Make sure exponent by itself

$7^{x+2} = 410$

② take log or ln this case doesn't matter

$\ln 7^{x+2} = \ln 410$

③ use prop above

$$\frac{(x+2) \cdot \ln 7}{\ln 7} = \frac{(\ln 410)}{(\ln 7)}$$

Must use calc divide by ln 7

$$x + 2 = 3.0917$$
$$-2 \quad\quad -2$$

$\boxed{x = 1.09}$

Now you need to remember $2^4 = 16$ Exponent form means $\log_2 16 = 4$ log form

Objective #3: Use exponential form to solve logarithmic equations.

✔ Solved Problem #3

3a. Solve: $4\ln(3x) = 8$.

$$4\ln(3x) = 8$$
$$\ln(3x) = 2$$
$$e^2 = 3x$$
$$\frac{e^2}{3} = x$$

Check:

$$4\ln(3x) = 8$$
$$4\ln\left[3\left(\frac{e^2}{3}\right)\right] = 8$$
$$4\ln(e^2) = 8$$
$$4 \cdot 2 = 8$$
$$8 = 8, \text{ true}$$

The solution set is $\left\{\frac{e^2}{3}\right\}$.

Pencil Problem #3

3a. Solve: $\log_4(x+5) = 3$.

this is log form so write it in exponential form

$4^3 = x+5$

now just use Algebra to solve

$64 = x+5$
$-5 \quad -5$
$\boxed{59 = x}$

Properties

times $\log_b x + \log_b y = \log_b x \cdot y$

divide $\log_b x - \log_b y = \log_b \frac{x}{y}$

Exponent $\log_b x^m = m \cdot \log_b x$

3b. Solve: $\log x + \log(x-3) = 1$.

$$\log x + \log(x-3) = 1$$
$$\log(x^2 - 3x) = 1$$
$$10^1 = x^2 - 3x$$
$$0 = x^2 - 3x - 10$$
$$0 = (x+2)(x-5)$$

$x+2 = 0$ or $x-5 = 0$

$x = -2$ $\quad x = 5$

Check -2:

$\log x + \log(x-3) = 1$

$\log(-2) + \log(-2-3) = 1$

-2 does not check.

Check 5:

$\log x + \log(x-3) = 1$

$\log 5 + \log(5-3) = 1$

$\log 5 + \log 2 = 1$

$\log 10 = 1$

$1 = 1$, true

The solution set is $\{5\}$.

3b. Solve: $\log_5 x + \log_5(4x-1) = 1$.

need to use properties to condense logs first same base

$\log_5 x(4x-1) = 1$

$\log_5 4x^2 - x = 1$

now write in exponential form

$5^1 = 4x^2 - x$
$-5 \qquad -5$

$0 = 4x^2 - x - 5$ $\quad -20 \quad -5, +4 \quad -1$

$\boxed{4x^2 + 4x} - 5x - 5 = 0$

box circle

$4x(x+1) - 5(x+1)$

$(4x-5)(x+1)$

$x = -1$ extraneous

$x = \frac{5}{4}$ ★

now check for extraneous

Check $x = -1$ No

$\log_5 -1 + \log_5 -5 = 1$

Can't take $5^{\square} = -1$ so $\frac{5}{4}$ works

Check

$\log_5 \frac{5}{4} + \log_5 (4 \cdot \frac{5}{4} - 1) = 1$

$\log_5 \frac{5}{4} \cdot 4 = 1$

$\log_5 5 = 1$ yes

$\frac{5}{4} = x$ only

College Algebra 6e

Objective #4: Use the one-to-one property of logarithms to solve logarithmic equations.

✔ Solved Problem #4

4. Solve: $\ln(x-3)=\ln(7x-23)-\ln(x+1)$

$$\ln(x-3)=\ln(7x-23)-\ln(x+1)$$
$$\ln(x-3)=\ln\left(\frac{7x-23}{x+1}\right)$$
$$x-3=\frac{7x-23}{x+1}$$
$$(x+1)(x-3)=(x+1)\frac{7x-23}{x+1}$$
$$x^2-2x-3=7x-23$$
$$x^2-9x+20=0$$
$$(x-4)(x-5)=0$$
$$x-4=0 \quad \text{or} \quad x-5=0$$
$$x=4 \qquad\qquad x=5$$

Check 4:

$$\ln(x-3)=\ln(7x-23)-\ln(x+1)$$
$$\ln(4-3)=\ln(7\cdot 4-23)-\ln(4+1)$$
$$\ln 1=\ln 5-\ln 5$$
$$0=0, \text{ true}$$

Check 5:

$$\ln(x-3)=\ln(7x-23)-\ln(x+1)$$
$$\ln(5-3)=\ln(7\cdot 5-23)-\ln(5+1)$$
$$\ln 2=\ln 12-\ln 6$$
$$\ln 2=\ln\left(\frac{12}{6}\right)$$
$$\ln 2=\ln 2, \text{ true}$$

The solution set is $\{4,5\}$.

Pencil Problem #4

4. Solve: $\log(x+4)-\log 2=\log(5x+1)$

Combine using properties

$$\log(X+4)-\log 2=\log 5X+1$$
$$\log\frac{X+4}{2}=\log 5X+1$$
$$\log\boxed{\text{this}}=\log\boxed{\text{that}}$$

guess what this = that

$$\frac{X+4}{2}=\frac{5X+1(\cdot 2)}{1(\cdot 2)}$$

bust butt

$$\frac{X+4}{2}=\frac{10X+2}{2}$$
$$X+4=10X+2$$
$$-X-2 \quad -X \quad -2$$
$$2=9X$$
$$\boxed{\frac{2}{9}=X}$$

Section 4.4

Objective #5: Solve applied problems involving exponential and logarithmic equations.

✔ Solved Problem #5

5a. Medical research indicates that the risk of having a car accident increases exponentially as the concentration of alcohol in the blood increases. The risk is modeled by $R = 6e^{12.77x}$ where x is the blood alcohol concentration and R, given as a percent, is the risk of having a car accident. What blood alcohol concentration corresponds to a 7% risk of a car accident?

$$R = 6e^{12.77x}$$
$$6e^{12.77x} = 7$$
$$e^{12.77x} = \frac{7}{6}$$
$$\ln e^{12.77x} = \ln\frac{7}{6}$$
$$12.77x = \ln\frac{7}{6}$$
$$x = \frac{\ln\frac{7}{6}}{12.77}$$
$$x \approx 0.01$$

For a blood alcohol concentration of 0.01, the risk of a car accident is 7%.

5b. How long, to the nearest tenth of a year, will it take \$1000 to grow to \$3600 at 8% annual interest compounded quarterly?

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$3600 = 1000\left(1+\frac{0.08}{4}\right)^{4t}$$
$$3.6 = 1.02^{4t}$$
$$1.02^{4t} = 3.6$$
$$\ln 1.02^{4t} = \ln 3.6$$
$$4t \ln 1.02 = \ln 3.6$$
$$t = \frac{\ln 3.6}{4\ln 1.02}$$
$$t \approx 16.2$$

After approximately 16.2 years, the \$1000 will grow to \$3600.

✎ Pencil Problem #5

typo should be x

5a. The formula $A = 37.3e^{0.0095x}$ models the population of California, A, in millions, t years after 2010. When will the population of California reach 40 million?

$$40 = 37.3e^{.0095x}$$
$$\frac{40}{37.3} = \frac{37.3e^{.0095x}}{37.3}$$

① First get exponent by itself

$$1.0724 = e^{.0095x}$$

② Now take ln of both side

$$\ln(1.0724) = \ln e^{.0095x}$$
$$\ln(1.0724) = .0095x \cdot \ln e$$

③ Use prop

equals 1

$$\frac{\ln(1.0724)}{.0095} = \frac{.0095x}{.0095}$$
$$7.36 = x \text{ years after 2010}$$

So In the year 2017 and 4 months the population in california will reach 40 million.

5b. How long, to the nearest tenth of a year, will it take \$8000 to grow to \$16,000 at 8% annual interest compounded continuously?

$.08 = r$ $\boxed{\text{looking for } t}$

$$A = Pe^{rt}$$
$$\frac{16000}{8000} = \frac{8000e^{.08t}}{8000}$$

First get $e^{.08t}$ by itself

take ln of both side

$$2 = e^{.08t}$$
$$\ln 2 = \ln e^{.08t}$$
$$\ln 2 = .08t \cdot \ln e$$

this is 1

$$\frac{\ln 2}{.08} = \frac{.08t}{.08}$$
$$t = 8.66$$

So in 8.66 Years your Money will grow from \$8000 to \$16000.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $\{a\}$ *(4.4 #7)* **1b.** $\{13\}$ *(4.4 #19)*

2a. $\{\log 3.91 \approx 0.59\}$ *(4.4 #23)*

2b. $\left\{\frac{\ln 410}{\ln 7} - 2 \approx 1.09\right\}$ *(4.4 #37)*

3a. $\{59\}$ *(4.4 #53)*

3b. $\left\{\frac{5}{4}\right\}$; note: reject -1 *(4.4 #67)*

4. $\left\{\frac{2}{9}\right\}$ *(4.4 #83)*

5a. 2017 *(4.4 #103b)*

5b. 8.7 years *(4.4 #111)*