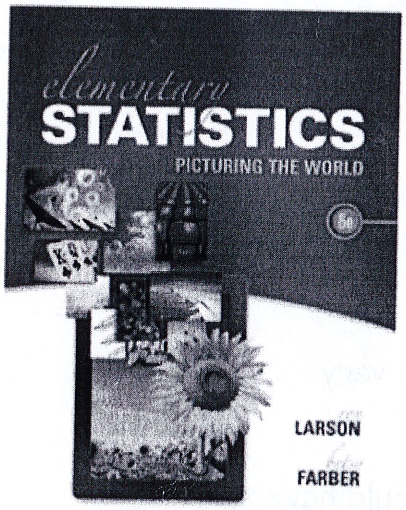
Elementary Statistics: Picturing The World

Sixth Edition



Chapter 7

Hypothesis Testing with One Sample

7.1 pg. 359 11-15 odd,21,22,23,26,29,30,37,41,42,44,47,48,50

7.2 pg. 373 3,5,7,10,12,14,16,18,19,21,24,25,27,30,31,39

7.3 pg. 383 3,5,7,11,12,14,15,21,22,23,27

7.4 pg. 391 3,8,9-11,15 OR 16,17

7.5 pg. 400 5,7,9,12,13,15,18,19,21,23

Chapter 7: Hypothesis Testing with One Sample

7-1: Introduction to Hypothesis Testing

In a nutshell: Hypothesis testing looks at if a statement seems likely to be false or not.

For instance: An auto company may claim their new cars get 50 mpg. You wonder if this is really true. You can't test all the new cars though, so now what?

- First you take a sample of 30 cars. You find the mean mpg is 47 with a standard deviation of 5.5. This is NOT exactly 50, but we wouldn't expect it to be. How different is it though from their claim?

- WE ASSUME THEY ARE CORRECT. Then we look at what would be weird values (outside of 2 standard deviations) from their claim about the mean being 50.

If 47 turns out to be more than 2s.d. away, we would be very suspicious about their claim.

If 47 was closer than 2 s.d. from the mean of 50, we would have to assume they might be right.

Since we assume they are truthful, the null hypothesis always includes an equality. The other option, that they are wrong, contains no equality and is the complement of their claim (the opposite of their statement).

DEFINITION

1. A **null hypothesis** H_0 is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$, or $\geq$.
2. The **alternative hypothesis** H_a is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and it contains a statement of strict inequality, such as $>$, $\neq$, or $<$.

H_0 is read as "H subzero" or "H naught" and H_a is read as "H sub-a."

We can write null and alternatives for any parameter - μ p σ, σ^2

Ho: Contain =
Ha: opposit

7-1: Continued

Verbal Statement H_0 The mean is . . .	Mathematical Statements	Verbal Statement H_a The mean is . . .
. . . greater than or equal to k. . . . at least k. . . . not less than k.	$\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$	. . . less than k. . . . below k. . . . fewer than k.
. . . less than or equal to k. . . . at most k. . . . not more than k.	$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$	. . . greater than k. . . . above k. . . . more than k.
. . . equal to k. . . . k. . . . exactly k.	$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$	. . . not equal to k. . . . different from k. . . . not k.

$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$ $\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$ $\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$

Example 1: Stating Null/Alternative Hypotheses

Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

$H_0: p = .61$ (Claim)
$H_a: p \neq .61$

2. A car dealership announces that the mean time for an oil change is less than 15 minutes.

$H_0: \mu \geq 15$
$H_a: \mu < 15$ Claim

3. A company advertises that the mean life of its furnaces is more than 18 years.

$H_0: \mu \leq 18$
$H_a: \mu > 18$ Claim

EXAMPLE 2

Identifying Type I and Type II Errors

The USDA limit for salmonella contamination for chicken is 20%. A meat inspector reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which error is more serious?

$H_0: p \leq .20$
$H_a: p > .20$ (claim)

Type I will be when the null $p \leq .20$ is rejected so the USDA says it is Not less than 20% contamination

Type II error the USDA doesn't reject the null and they actually do have more than in the use 20%

TYPES OF ERRORS AND LEVEL OF SIGNIFICANCE

No matter which hypothesis represents the claim, you always begin a hypothesis test by assuming that the equality condition in the null hypothesis is true. So, when you perform a hypothesis test, you make one of two decisions:

1. reject the null hypothesis

or

2. fail to reject the null hypothesis.

DEFINITION

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

Convicted when Innocent

Guilty but let go.

Court Case
Ho: Innocent Null
Ha: ≠ Guilty Alter

next page

The following table shows the four possible outcomes of a hypothesis test.

	Truth of H_0	
Decision	H_0 is true. Innocent	H_0 is false. Guilty
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

Type I
Ho: = Cancer reject
Ha: ≠ Cancer
Reject say don't Cancer but do true

Put an innocent person behind bars

Let a guilty person go free

next page

7-1 continued... Errors, levels of significance

Types of errors:

Level of significance: All samples will vary some from the population parameter (remember our errors from CIs?). How often will you reject the null hypothesis when it is actually true (false positive)? 5% of the time? 1%? 10%? This is your LEVEL OF SIGNIFICANCE.

DEFINITION

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a type I error. It is denoted by α, the lowercase Greek letter alpha.

	Truth About Defendant	
Verdict	**Innocent**	**Guilty**
Not guilty	Justice	Type II error
Guilty	Type I error	Justice

1. A carefully worded accusation is written.
2. The defendant is assumed innocent (H_0) until proven guilty. The burden of proof lies with the prosecution. If the evidence is not strong enough, then there is no conviction. A "not guilty" verdict does not prove that a defendant is innocent.
3. The evidence needs to be conclusive beyond a reasonable doubt. The system assumes that more harm is done by convicting the innocent (type I error) than by not convicting the guilty (type II error).

What we want is to be able to compare our test statistic's value versus the value that is claimed to be true, which means we need to convert them into our standard distributions (z-scores, t-scores, or chi-squared scores).

Same as CIs Remember

Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 7.2, σ known), t (Section 7.3, σ unknown)
p	$\hat{p}$	z (Section 7.4)
σ^2	s^2	χ^2 (Section 7.5)

We then compare the P-value (probability) of getting a sample value as extreme or more extreme than the unusual ranges of our claim's distribution.

DEFINITION

If the null hypothesis is true, a ***P*-value** (or **probability value**) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

Reminder: (Summary of all of Chapter 6!)

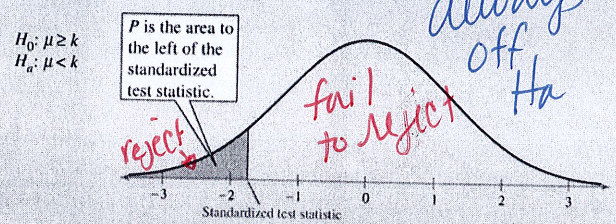
7-1 continued... Type of test, P-values 2 ways to prove

The *P*-value of a hypothesis test depends on the nature of the test. There are three types of hypothesis tests—**left-tailed, right-tailed,** and **two-tailed.** The type of test depends on the location of the region of the sampling distribution that favors a rejection of H_0. This region is indicated by the alternative hypothesis.

Find Rejection Region ✓method

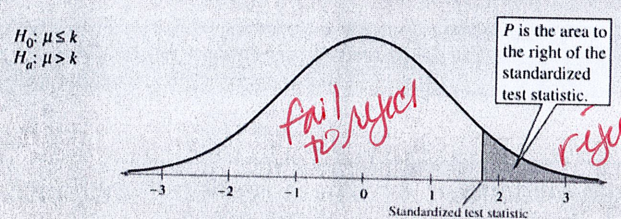
DEFINITION

1. If the alternative hypothesis H_a contains the less-than inequality symbol ($<$), then the hypothesis test is a **left-tailed test.**



Left-Tailed Test

2. If the alternative hypothesis H_a contains the greater-than inequality symbol ($>$), then the hypothesis test is a **right-tailed test.**

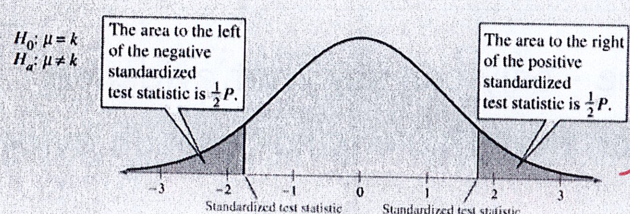


Right-Tailed Test

next pg →

The smaller the P-value of the test the more evidence there is to reject the null hypothesis. A very small P-value indicated an unusual event.

3. If the alternative hypothesis H_a contains the not-equal-to symbol ($\neq$), then the hypothesis test is a **two-tailed test.** In a two-tailed test, each tail has an area of $\frac{1}{2}P$.



Two-Tailed Test

Remember even a very low P-value doesn't constitute proof that the null is false

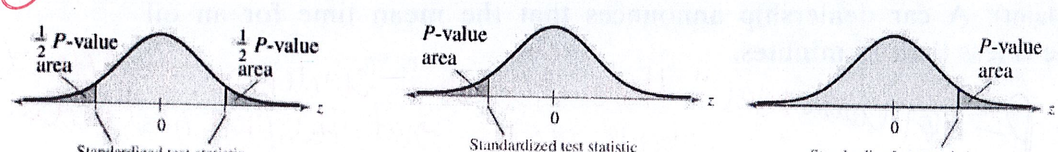
Example 3: Type of Hypothesis Test?

Identifying the Nature of a Hypothesis Test

For each claim, state H_0 and H_a in words and in symbols. Then determine whether the hypothesis test is a left-tailed test, right-tailed test, or two-tailed test. Sketch a normal sampling distribution and shade the area for the *P*-value.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. A car dealership announces that the mean time for an oil change is less than 15 minutes.
3. A company advertises that the mean life of its furnaces is more than 18 years.

$H_0: \mu \geq 15$ mean time is greater than or equal to 15.
$H_a: \mu < 15$ (claim) mean time is less than 15



1) $H_0: p = .61$ null at least one act is 61%
$H_a: p \neq .61$ claim at least one act is Not 61%

③. $H_0: \mu \leq 18$ less than or equal to 18
$H_a: \mu > 18$ More than 18 years

P-value Way

7.1 Continued

DECISION RULE BASED ON P-VALUE

To use a P-value to make a conclusion in a hypothesis test, compare the P-value with α.

1. If $P \le \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

The smaller the P-value of the test, the more evidence there is to reject the null hypothesis. A very small P-value indicates an unusual event. Remember, however, that even a very low P-value does not constitute proof that the null hypothesis is false, only that it is probably false.

	Claim	
Decision	**Claim is H_0.**	**Claim is H_a.**
Reject H_0.	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

You can't prove things true - only can say that evidence indicates that it is false, or that there is not enough evidence to say that it is false!

Reject $H_0: \mu \ge 15$

There is enough evidence to support the $H_a: \mu < 15$ claim

dealers claim the oil change takes. less than 15 min

fail to Reject $H_0: \mu \ge 15$

there is not enough evidence to support the dealers claim less than 15 min.

EXAMPLE 4

Interpreting a Decision

You perform a hypothesis test for each claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

1. H_0 (Claim): A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. H_a (Claim): A car dealership announces that the mean time for an oil change is less than 15 minutes.

① $H_0: p = .61$ Claim

$H_a: p \ne .61$

Reject "there is enough evidence to reject the schools claim that the prop of students in atleast one act is 61%

fail to Reject: "there is Not enough evidence to reject the schools claim that the true prop of students in atleast one act is 61%

7-1 continued... Strategies

Writing hypotheses depends on which side of the argument you believe. You want to be able to reject the null, so your belief should be in the alternative hypothesis.

Example 5: Writing the hypotheses

A medical research team is investigating the benefits of a new surgical treatment. One of the claims is that the mean recovery time for patients after the new treatment is less than 96 hours.

1. Write the hypotheses as if you are on the research team and want to support the claim. $H_0: \mu \geq 96$
   $H_a: \mu < 96$ claim

   Want to Support claim needs to be in Ha

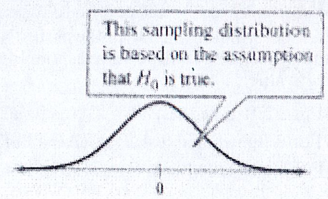
2. Write the hypotheses as if you are on an opposing team and want to reject the claim. $H_0: \mu \leq 96$ (Claim) ← You want this to be rejected
   $H_a: \mu > 96$

   needs to be in Ho

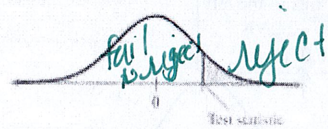
GENERAL STEPS - USE THROUGHOUT THIS CHAPTER!!!

STEPS FOR HYPOTHESIS TESTING

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
   H_0: ? H_a: ?
2. Specify the level of significance.
   α = ?
   ← you decide Most time use $\alpha = .05$
3. Determine the standardized sampling distribution and sketch its graph.



4. Calculate the test statistic and its standardized value. Add it to your sketch.



5. Find the *P*-value.
6. Use the following decision rule.

Is the *P*-value less than or equal to the level of significance? → No → Fail to reject H_0.

↓ Yes

Reject H_0.

$P \leq \alpha$ Reject Ho

→ There is enough evidence to reject the claim if your claim is in Ho

→ If your claim is in Ha you support your claim

$P > \alpha$ Fail to reject

→ there is not enough evidence to support your claim, if in Ho.

→ if your claim is in Ha there is not enough evidence to support your claim Ha

7. Write a statement to interpret the decision in the context of the original claim.

⇨ pg 359: 11-16,21,22, 23,26,29,30, 37, 41,42,44, 47, 49,50

Chapter 7: Hypothesis Testing with One Sample

7-2: Hypothesis Tests when you know σ

One parameter we often want to do a hypothesis test on is the mean. Just like CIs, there are different methods if the sample is ~~over 30 or under 30~~. If we know σ or not

Reminder from yesterday:

DECISION RULE BASED ON P-VALUE

To use a P-value to make a conclusion in a hypothesis test, compare the P-value with α.

1. If $P \le \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

Example 1: Interpreting a P-value

The P-value for a hypothesis test is P=0.0237. What is your decision if the level of significance is...

1) $\alpha = 0.05$ $P < .05$ Reject Null

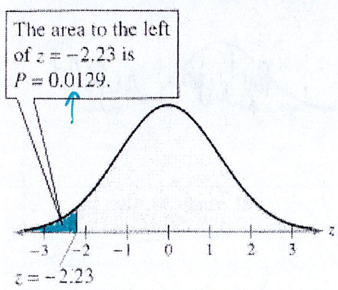
2) $\alpha = 0.01$ $P > .01$ fail to reject the null

Finding those P-values is going to be very important, clearly! Use the tables (or for normal distributions, normalcdf(_, _)) ← remember ch5

FINDING THE P-VALUE FOR A HYPOTHESIS TEST

After determining the hypothesis test's standardized test statistic and the standardized test statistic's corresponding area, do one of the following to find the P-value.

a. For a left-tailed test, P = (Area in left tail).
b. For a right-tailed test, P = (Area in right tail).
c. For a two-tailed test, P = 2(Area in tail of standardized test statistic).



Left-Tailed Test

Example 2: Finding P-value for left-tailed tests

Find the P-value for a left-tailed hypothesis test with a test statistic of z=-2.23. Decide whether to reject the null if the level of significance is 0.01.

Draw a picture

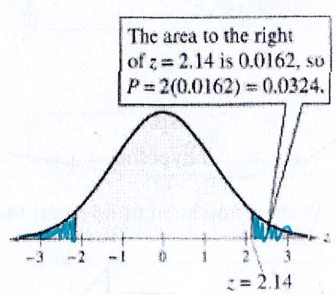
P = Area in tail

$p = .0129$

$p > \alpha$ fail to Reject Ho

Example 2: Finding P-value for two-tailed tests

Find the P-value for a two-tailed hypothesis test with a test statistic of z=2.14. Decide whether to reject the null if the level of significance is 0.05.



Two-Tailed Test

P = total Area

$2(.0162)$

$p = .0324$

$\boxed{P < \alpha \text{ Reject the } H_0}$