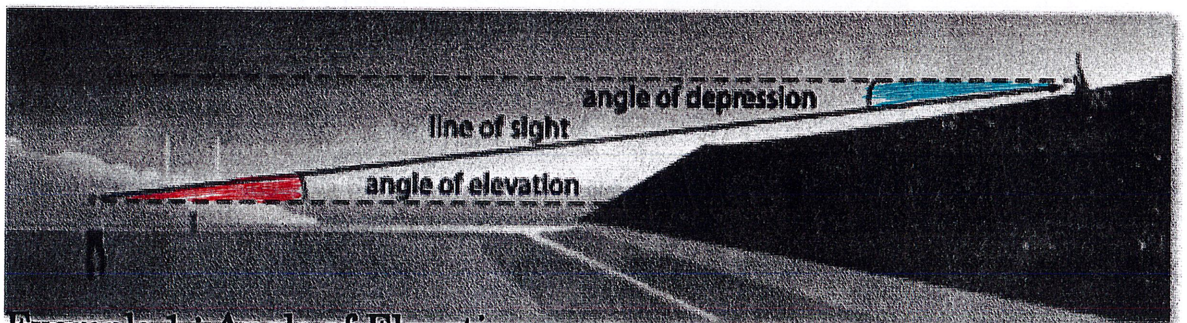
8.5 Angle of elevation and depression

Angle of elevation is formed by a horizontal line and an observer's line of sight to an object above the horizontal line.

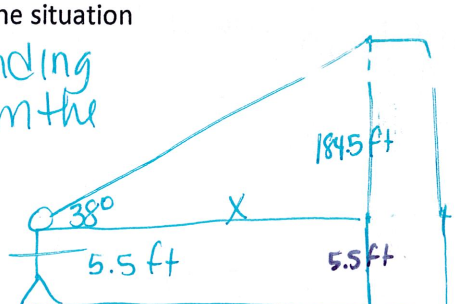
Angle of depression is the angle formed by a horizontal line and an observer's line of sight to an object below the horizontal line.



Example 1 : Angle of Elevation

Leah wants to see a castle in an amusement park. She sights the top of the castle at an angle of elevation of 38°. She knows that the castle is 190 feet tall. If Leah is 5.5 feet tall, how far is she from the castle to the nearest foot. Make a sketch to represent the situation

She is standing 236 ft from the castle.



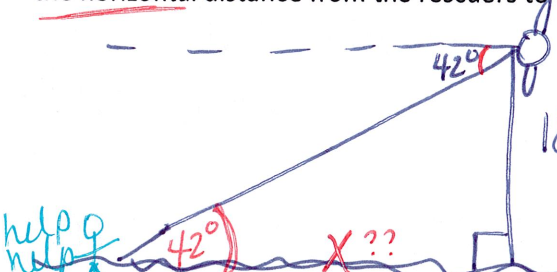
$\frac{\tan 38°}{1} = \frac{184.5}{x}$ get x by itself

$\frac{x \cdot \tan 38}{\tan 38} = \frac{184.5}{\tan 38°}$

$x = 236.15$

Example 2: Angle of Depression

A search and rescue team is airlifting people form the scene of a boating accident when they observe another person in need of help. If the angle of depression to this other period is 42° and the helicopter is 18 feet above the water, what is the horizontal distance from the rescuers to this person to the nearest foot.



$\frac{\tan 42}{1} = \frac{18}{x}$

$\frac{x \cdot \tan 42}{\tan 42} = \frac{18}{\tan 42}$

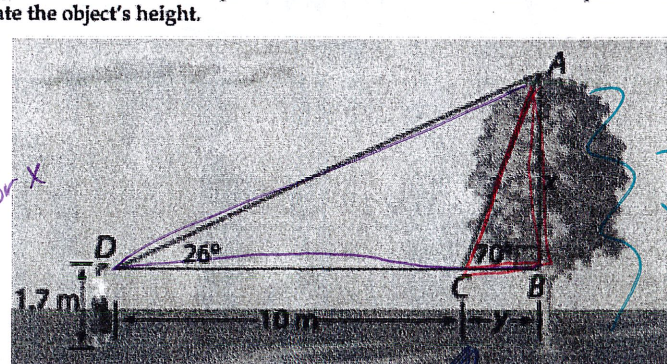
$x = 19.99$

20 ft

The horizontal distance the helicopter is from person is 20 ft.

2 Two Angles of Elevation or Depression Angles of elevation or depression to two different objects can be used to estimate the distance between those objects. Similarly, the angles from two different positions of observation to the same object can be used to estimate the object's height.

TREE REMOVAL To estimate the height of a tree she wants removed, Mrs. Long sights the tree's top at a 70° angle of elevation. She then steps back 10 meters and sights the top at a 26° angle. If Mrs. Long's line of sight is 1.7 meters above the ground, how tall is the tree to the nearest meter?



find x then add 1.7m

$\tan 70 = \frac{x}{y}$ Solve for x

$y \cdot \tan 70 = x$

Solve for x: $\tan 26 = \frac{x}{10+y}$

$(10+y) \tan 26 = x$

$y \tan 70 = (10+y) \tan 26$

$y \tan 70 = 10 \tan 26 + y \cdot \tan 26 \rightarrow$
$- y \tan 26 \qquad - y \cdot \tan 26$

$y \tan 70 - y \tan 26 = 10 \tan 26$

$y(\tan 70 - \tan 26) = 10 \tan 26$

Small triangle $\tan 70 = \frac{x}{y}$ big triangle $\tan 28 = \frac{x}{y+10}$

Solve both for x

$y \cdot \tan 70 = x$ $(y+10)\tan 28 = x$

now you can set them equal to eachother and solve because thy both $= x$

$y \tan 70 = (y+10)\tan 28$ Distribute

$y \tan 70 = y \tan 28 + 10 \tan 28$ get all y on same side

$-y \tan 28 \quad -y \tan 28$

$y \tan 70 - y \tan 28 = 10 \tan 28$ GCF factor out a y

$\frac{y(\tan 70 - \tan 28)}{\tan 70 - \tan 28} = \frac{10 \tan 28}{\tan 70 - \tan 28}$ divide by tan70-tan28 to get y by itself

$y = \frac{10 \tan 26}{\tan 70 - \tan 28}$ $\frac{4.877}{2.2157}$

now use calc $y = 2.16$

$y \cdot \tan 70 = x$

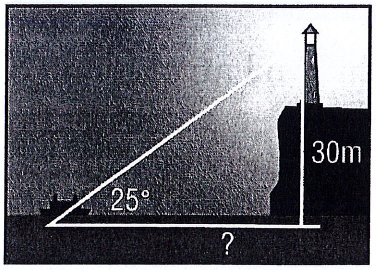
$2.16 \cdot \tan 70 = x$

$\boxed{x = 5.93}$ So now add 1.7

The height of the tree is 7.63 meters

8.5 Angles of Elevation and Depression

1. **LIGHTHOUSES** Sailors on a ship at sea spot the light from a lighthouse. The angle of elevation to the light is 25°.



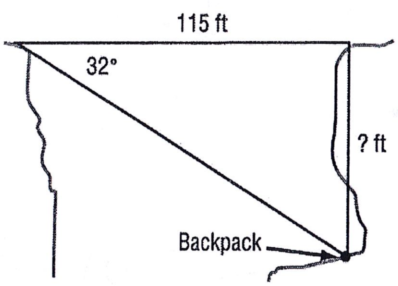
$\frac{\tan 25}{1} = \frac{30}{X}$ $\quad X = 64.3$

$X \cdot \tan 25 = 30$ $\quad$ The

The light of the lighthouse is 30 meters above sea level. How far from the shore is the ship? Round your answer to the nearest meter.

The Ship is 64 meters from the Shore.

2. **RESCUE** A hiker dropped his backpack over one side of a canyon onto a ledge below. Because of the shape of the cliff, he could not see exactly where it landed.



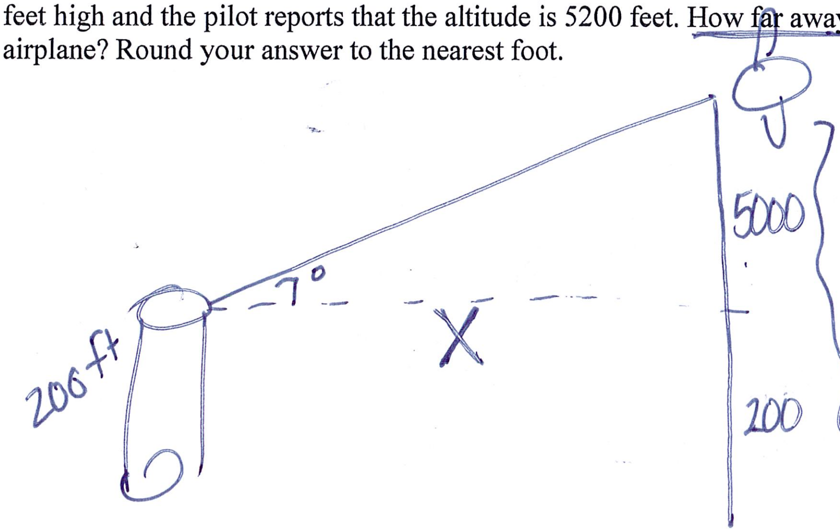
$\tan 32 = \frac{X}{115}$

$X = 115 \cdot \tan 32°$

From the other side, the park ranger reports that the angle of depression to the backpack is 32°. If the width of the canyon is 115 feet, how far down did the backpack fall? Round your answer to the nearest foot.

The backpack is 72 feet below where the hiker dropped it.

3. **AIRPLANES** The angle of elevation to an airplane viewed from the control tower at an airport is 7°. The tower is 200 feet high and the pilot reports that the altitude is 5200 feet. How far away from the control tower on a straight line is the airplane? Round your answer to the nearest foot.



5200ft

$\tan 7 = \frac{5000}{X}$

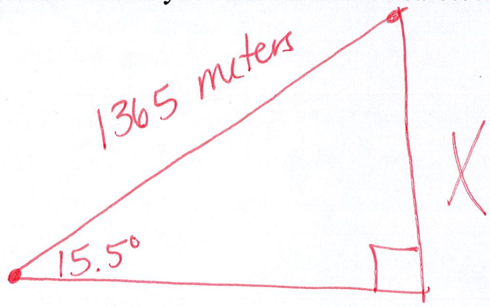
$\frac{X \cdot \tan 7}{\tan 7} = \frac{5000}{\tan 7}$

$X = 40722$

The airplane is 40,722 ft from the tower.

8.5 Angles of Elevation and Depression

PEAK TRAM The Peak Tram in Hong Kong connects two terminals, one at the base of a mountain, and the other at the summit. The angle of elevation of the upper terminal from the lower terminal is about 15.5°. The distance on a line between the two terminals is about 1365 meters. About how much higher above sea level is the upper terminal compared to the lower terminal? Round your answer to the nearest meter.

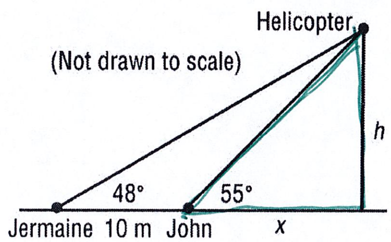


$\sin 15.5° = \frac{X}{1365}$

$X = 364.8$

The upper terminal is 365 meters above the lower terminal.

5. HELICOPTERS Jermaine and John are watching a helicopter hover above the ground.



Solve for h

John

$\tan 55 = \frac{h}{x}$

$h = x \cdot \tan 55$

Jermaine

$\tan 48 = \frac{h}{10+x}$

$(10+x)\tan 48 = h$

Jermaine and John are standing 10 meters apart.

a. Find two different expressions that can be used to find the *h*, height of the helicopter.

John $h = x \cdot \tan 55$ Jermaine $h = (10+x)\tan 48$

$h = 10\tan 48 + x\tan 48$

b. Equate the two expressions you found for Exercise a to solve for *x*. Round your answer to the nearest hundredth.

$x \cdot \tan 55 = 10\tan 48 + x\tan 48$

$-x\tan 48 \qquad -x\tan 48$

get all x on same side

c. How high above the ground is the helicopter? Round your answer to the nearest hundredth.

$x\tan 55 - x\tan 48 = 10\tan 48$ factor out an x

$\frac{x(\tan 55 - \tan 48)}{\tan 55 - \tan 48} = \frac{10\tan 48}{\tan 55 - \tan 48}$ divide

$x = \frac{10\tan 48}{\tan 55 - \tan 48} \approx 34.97$

x so now

$h = x \cdot \tan 55$

$h = 34.97 \tan 55$

The helicopter is 49.95 meters above the ground.