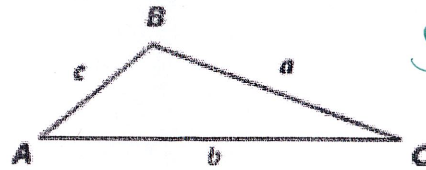
8.6 Law of Sines and Law of Cosines

Theorem 8.10 Law of Sines

If $\triangle ABC$ has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

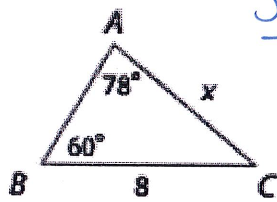


$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$

Example 1 Law of Sines AAS

GuidedPractice

1A.

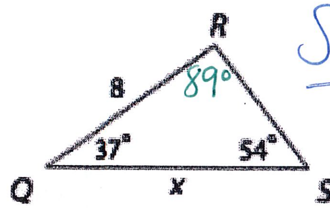


$$\frac{\sin 60}{x} = \frac{\sin 78}{8}$$

$$\frac{x \cdot \sin 78}{\sin 78} = \frac{8 \cdot \sin 60}{\sin 78}$$

$$\boxed{x = 7.08}$$

1B.



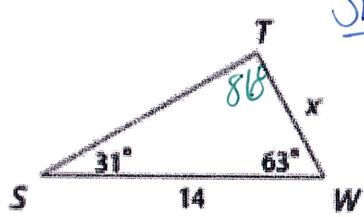
$$\frac{\sin 54}{8} = \frac{\sin 89}{x}$$

$$\frac{x \cdot \sin 54}{\sin 54} = \frac{8 \cdot \sin 89}{\sin 54}$$

$$\boxed{x = 9.89}$$

Example 2 Law of Sines ASA

2A.

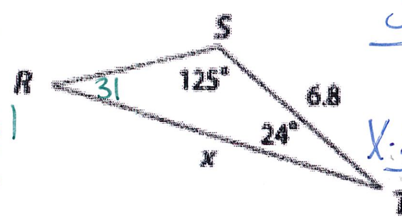


$$\frac{\sin 31}{x} = \frac{\sin 86°}{14}$$

$$\frac{x \cdot \sin 86}{\sin 86} = \frac{14 \sin 31}{\sin 86}$$

$$\boxed{x = 7.23}$$

2B.



$$\frac{\sin 125}{x} = \frac{\sin 31}{6.8}$$

$$\frac{x \cdot \sin 31}{\sin 31} = \frac{6.8 \sin 125}{\sin 31}$$

$$\boxed{x = 10.82}$$

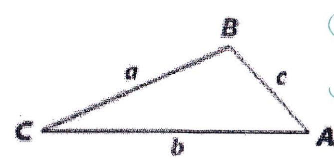
Theorem 8.11 Law of Cosines

If $\triangle ABC$ has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

$a^2 = b^2 + c^2 - 2bc \cos A$,

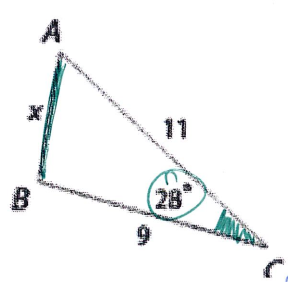
$b^2 = a^2 + c^2 - 2ac \cos B$, and

$c^2 = a^2 + b^2 - 2ab \cos C$.



SSS
SAS

Example 3 Law of Cosines SAS



$$x^2 = 9^2 + 11^2 - 2(9)(11)\cos 28°$$

$$x^2 = 81 + 121 - 198 \cos 28$$

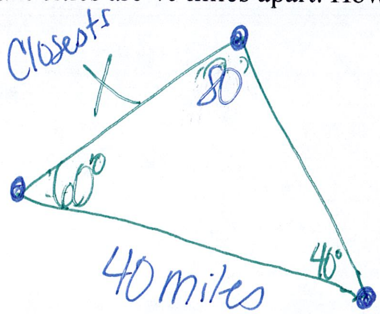
$$x^2 = 202 - 198 \cos 28$$

$$\sqrt{x^2} = \sqrt{27.176}$$

$$\boxed{x = 5.213}$$

8.6 Law of Sines and Law of Cosines

MAPS Three cities form the vertices of a triangle. The angles of the triangle are 40°, 60°, and 80°. The two most distant cities are 40 miles apart. How close are the two closest cities? Round your answer to the nearest tenth of a mile.



ASA So law of Sines

$$\frac{\sin 80}{40} = \frac{\sin 40}{X}$$

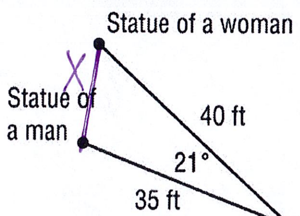
$$\frac{X \cdot \sin 80}{\sin 80} = \frac{40 \cdot \sin 40}{\sin 80}$$

$X = 26.11$

The 2 Cities that are Closest are 26.1 miles apart.

3. STATUES Gail was visiting an art gallery. In one room, she stood so that she had a view of two statues, one of a man, and the other of a woman. She was 40 feet from the statue of the woman, and 35 feet from the statue of the man. The angle created by the lines of sight to the two statues was 21°. What is the distance between the two statues? Round your answer to the nearest tenth.

Law of Cosines



$X^2 = 35^2 + 40^2 - 2(35)(40)\cos 21$

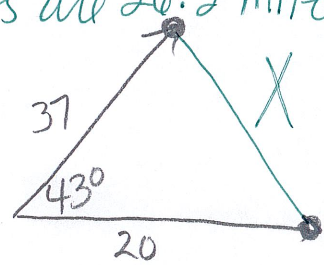
$X^2 = 2825 - 2800\cos 21$

$X^2 = 210.97$

The two Statues are 14.5 ft apart.

4. CARS Two cars start moving from the same location. They head straight, but in different directions. The angle between where they are heading is 43°. The first car travels 20 miles and the second car travels 37 miles. How far apart are the two cars? Round your answer to the nearest tenth.

The two Cars are 26.2 Miles apart.



Law of Cosines

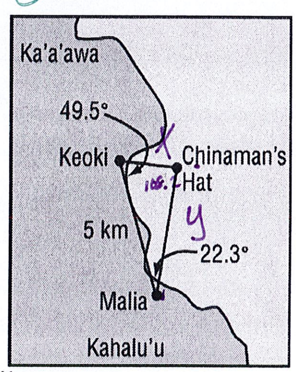
$X^2 = 20^2 + 37^2 - 2(20)(37)\cos 43$

$X^2 = 1769 - 1480\cos 43$

$X^2 = 686.6$

$X = 26.2$

5. ISLANDS Oahu is a Hawaiian Island. Off of the coast of Oahu, there is a very tiny island known as Chinaman's Hat. Keoki and Malia are observing Chinaman's Hat from locations 5 kilometers apart. Use the information in the figure to answer the following questions.



a. How far is Keoki from Chinaman's Hat? Round your answer to the nearest tenth of a kilometer.

$$\frac{\sin 22.3}{X} = \frac{\sin 108.2}{5}$$

$$\frac{X \cdot \sin 108.2}{\sin 108.2} = \frac{5 \sin 22.3}{\sin 108.2}$$

$X = 1.997$

Keoki is 2 km from Chinaman's Hat.

b. How far is Malia from Chinaman's Hat? Round your answer to the nearest tenth of a kilometer.

$$\frac{\sin 49.5}{y} = \frac{\sin 108.2}{5}$$

$$\frac{5 \sin 49.5}{\sin 108.2} = \frac{y \cdot \sin 108.2}{\sin 108.2}$$

$4.00 = y$

Malia is 4 km from Chinaman's Hat.