

Section 1.2

Check Point Exercises

$$\begin{aligned}
 1. \quad & 4x + 5 = 29 \\
 & 4x + 5 - 5 = 29 - 5 \\
 & 4x = 24 \\
 & \frac{4x}{4} = \frac{24}{4} \\
 & x = 6
 \end{aligned}$$

Check:

$$4x + 5 = 29$$

$$4(6) + 5 = 29$$

$$24 + 5 = 29$$

$$29 = 29 \text{ true}$$

The solution set is $\{6\}$.

$$7. \quad 4x - 7 = 4(x - 1) + 3$$

$$4x - 7 = 4(x - 1) + 3$$

$$4x - 7 = 4x - 4 + 3$$

$$4x - 7 = 4x - 1$$

$$-7 = -1$$

The original equation is equivalent to the statement

 $-7 = -1$, which is false for every value of x .The solution set is the empty set, $\emptyset$.

The equation is an inconsistent equation.

$$13. \quad 16 = 3(x - 1) - (x - 7)$$

$$16 = 3x - 3 - x + 7$$

$$16 = 2x + 4$$

$$12 = 2x$$

$$6 = x$$

The solution set is $\{6\}$.

Check:

$$16 = 3(6 - 1) - (6 - 7)$$

$$16 = 3(5) - (-1)$$

$$16 = 15 + 1$$

$$16 = 16$$

$$19. \quad 20 - \frac{x}{3} = \frac{x}{2}$$

$$6 \left[20 - \frac{x}{3} = \frac{x}{2} \right]$$

$$120 - 2x = 3x$$

$$120 = 3x + 2x$$

$$120 = 5x$$

$$x = \frac{120}{5}$$

$$x = 24$$

The solution set is $\{24\}$.

$$25. \quad \frac{x+3}{6} = \frac{3}{8} + \frac{x-5}{4}$$

$$24 \left[\frac{x+3}{6} = \frac{3}{8} + \frac{x-5}{4} \right]$$

$$4x + 12 = 9 + 6x - 30$$

$$4x - 6x = -21 - 12$$

$$-2x = -33$$

$$x = \frac{33}{2}$$

The solution set is $\left\{ \frac{33}{2} \right\}$.

$$31. \quad a. \quad \frac{4}{x} = \frac{5}{2x} + 3 \quad (x \neq 0)$$

$$b. \quad \frac{4}{x} = \frac{5}{2x} + 3$$

$$8 = 5 + 6x$$

$$3 = 6x$$

$$\frac{1}{2} = x$$

The solution set is $\left\{ \frac{1}{2} \right\}$.

$$37. \quad a. \quad \frac{x-2}{2x} + 1 = \frac{x+1}{x} \quad (x \neq 0)$$

$$b. \quad \frac{x-2}{2x} + 1 = \frac{x+1}{x}$$

$$x - 2 + 2x = 2x + 2$$

$$x - 2 = 2$$

$$x = 4$$

The solution set is $\{4\}$.

$$43. \quad a. \quad \frac{3}{2x-2} + \frac{1}{2} = \frac{2}{x-1} \quad (x \neq 1)$$

$$b. \quad \frac{3}{2x-2} + \frac{1}{2} = \frac{2}{x-1}$$

$$\frac{3}{2(x-1)} + \frac{1}{2} = \frac{2}{x-1}$$

$$3 + 1(x-1) = 4$$

$$3 + x - 1 = 4$$

$$x = 2$$

The solution set is $\{2\}$.

$$45. \quad a. \quad \frac{3}{x+2} + \frac{2}{x-2} = \frac{8}{(x+2)(x-2)}; (x \neq -2, 2)$$

$$b. \quad \frac{3}{x+2} + \frac{2}{x-2} = \frac{8}{(x+2)(x-2)}$$

$$(x \neq 2, x \neq -2)$$

$$3(x-2) + 2(x+2) = 8$$

$$3x - 6 + 2x + 4 = 8$$

$$5x = 10$$

$$x = 2 \Rightarrow \text{no solution}$$

The solution set is the empty set, $\emptyset$.

$$49. \quad a. \quad \frac{1}{x-4} - \frac{5}{x+2} = \frac{6}{(x-4)(x+2)}; (x \neq -2, 4)$$

$$b. \quad \frac{1}{x-4} - \frac{5}{x+2} = \frac{6}{x^2 - 2x - 8}$$

$$\frac{1}{x-4} - \frac{5}{x+2} = \frac{6}{(x-4)(x+2)}$$

$$(x \neq 4, x \neq -2)$$

$$1(x+2) - 5(x-4) = 6$$

$$x + 2 - 5x + 20 = 6$$

$$-4x = -16$$

$$x = 4 \Rightarrow \text{no solution}$$

The solution set is the empty set, $\emptyset$.

53. Set $y_1 - y_2 = 1$.

$$\begin{aligned}\frac{x-3}{5} - \frac{x-5}{4} &= 1 \\ 20 \cdot \frac{x-3}{5} - 20 \cdot \frac{x-5}{4} &= 20 \cdot 1 \\ 4(x-3) - 5(x-5) &= 20 \\ 4x-12-5x+25 &= 20 \\ -x+13 &= 20 \\ -x &= 7 \\ x &= -7\end{aligned}$$

The solution set is $\{-7\}$.

65. $10x+3=8x+3$

$$2x+3=3$$

$$2x=0$$

$$x=0$$

The solution set $\{0\}$.

The given equation is a conditional equation.

77. $\frac{4}{x-2} + \frac{3}{x+5} = \frac{7}{(x+5)(x-2)}$

$$4(x+5) + 3(x-2) = 7$$

$$4x+20+3x-6=7$$

$$7x=-7$$

$$x=-1$$

The solution set is $\{-1\}$.

The given equation is a conditional equation.

59.

$$0 = \frac{x+6}{3x-12} - \frac{5}{x-4} - \frac{2}{3}$$

$$0 = \frac{x+6}{3(x-4)} - \frac{5}{x-4} - \frac{2}{3}$$

$$3(x-4) \cdot 0 = 3(x-4) \left(\frac{x+6}{3(x-4)} - \frac{5}{x-4} - \frac{2}{3} \right)$$

$$0 = \frac{3(x-4)(x+6)}{3(x-4)} - \frac{5 \cdot 3(x-4)}{x-4} - \frac{2 \cdot 3(x-4)}{3}$$

$$0 = (x+6) - 15 - 2(x-4)$$

$$0 = x+6-15-2x+8$$

$$0 = -x-1$$

$$x = -1$$

The solution set is $\{-1\}$.

71. $\frac{2}{x-2} = 3 + \frac{x}{x-2}$

$$2 = 3(x-2) + x$$

$$2 = 3x - 6 + x$$

$$-4x = -8$$

$$x = 2 \Rightarrow \text{no solution}$$

The solution set is the empty set, $\emptyset$.

The given equation is an inconsistent equation.

83. The equation is $-3(x-3) = 5(2-x)$, and the solution is $x = 0.5$.

85. Solve: $4(x-2)+2=4x-2(2-x)$

$$4x-8+2=4x-4+2x$$

$$4x-6=6x-4$$

$$-2x-6=-4$$

$$-2x=2$$

$$x=-1$$

Now, evaluate $x^2 - x$ for $x = -1$:

$$x^2 - x = (-1)^2 - (-1)$$

$$= 1 - (-1) = 1 + 1 = 2$$