Honors 2

Chapter 2: Reasoning and Proof

Day	Topic	Assignment
1	Classroom Expectations and Rules	Ticket Time Alg 1 Review talk about Boot Camp
2	2.1 Inductive Reasoning and Conj.	Pg. 94 55, 57, 65, 66, 68
3	2.2 Logic	Algebra Review WKSH
4	2.3 Conditional Statements	Pg. 111 19-51 EOO 59-61 69-72
5	2.4 Deductive Reasoning	Pg 121 17-20 24, 25, 31, 32
6	QUIZ 2.1 - 2.4	
7	Proof Example Game Day 2-5	2-5 WKSH
8	2.6 Algebraic Proofs	Pg. 141 23-26 You Pick 2 42-45
9	2.7 Proving Segment Relationships	Pg. 148 9-13 pick 2 23-26
10	2.8 Proving Angle Relationships	Pg 156 9, 11, 13, 20 36-39
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2-1 Study Guide and Intervention

Inductive Reasoning and Conjecture

Making Conjectures **Inductive reasoning** is reasoning that uses information from different examples to form a conclusion or statement called a Conjecture

Example 1: Write a conjecture about the next number in the sequence 1, 3, 9, 27, 81.

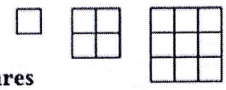
Look for a pattern:

Each number is a power of 3.

1	3	9	27	81	243
3^0	3^1	3^2	3^3	3^4	3^5

Conjecture: The next number will be 3^5 or 243.

Example 2: Write a conjecture about the number of small squares in the next figure.



Look for a pattern: The sides of the squares have measures 1, 2, and 3 units.

Conjecture: For the next figure, the side of the square will be 4 units, so the figure will have 16 small squares.

Exercises

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

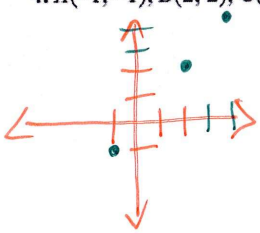
1. –5, 10, –20, 40, -80 Mult by -2 So -80 would be next

2. 1, 10, 100, 1000, 10000 add a zero or times by 10

3. 1, $\frac{6}{5}$, $\frac{7}{5}$, $\frac{8}{5}$ $\frac{5}{5}$, $\frac{6}{5}$, $\frac{7}{5}$, $\frac{8}{5}$, $\frac{9}{5}$ add $\frac{1}{5}$ each time

Write a conjecture about each value or geometric relationship.

4. $A(-1, -1)$, $B(2, 2)$, $C(4, 4)$ X = y



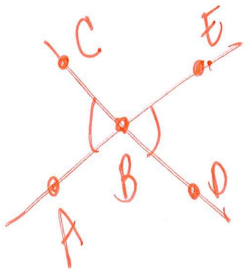
the y coordinate is equal to the x Collinear

5. ∠1 and ∠2 form a right angle.

∠1 & ∠2 Must be acute and ∠1 & ∠2 are complementary

6. ∠*ABC* and ∠*DBE* are vertical angles.

then ∠ABC ≅ ∠DBE



7. ∠*E* and ∠*F* are right angles.

therefore ∠E ≅ ∠F

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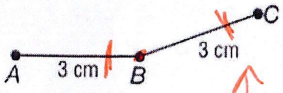
2-1 Study Guide and Intervention *(continued)*

Inductive Reasoning and Conjecture

Find Counterexamples A conjecture is false if there is even one situation in which the conjecture is Not true. The false example is called a **counterexample**.

Example: Find a counterexample to show the conjecture is false.

If $\overline{AB} \cong \overline{BC}$, then B is the midpoint of $\overline{AC}$.
Is it possible to draw a diagram with $\overline{AB} \cong \overline{BC}$ such that B is not the midpoint? This diagram is a counterexample because point B is not on $\overline{AC}$. The conjecture is false.



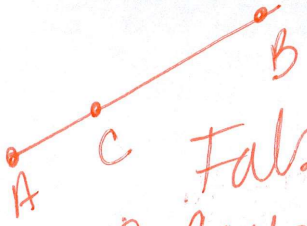
false because ↑ Counterexample

Exercises

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

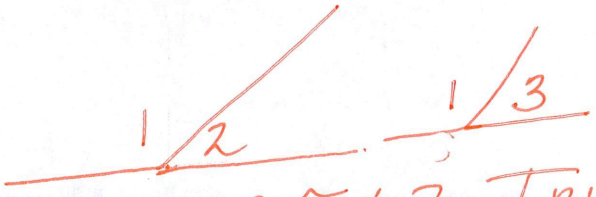
1. If points A, B, and C are collinear, then $AB + BC = AC$.

on same line



False
C could be between A & B

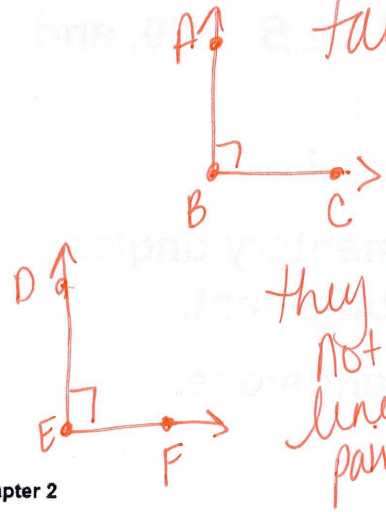
2. If $\angle R$ and $\angle S$ are supplementary, and $\angle R$ and $\angle T$ are supplementary, then $\angle T$ and $\angle S$ are congruent.



$\angle 3 \cong \angle 2$ TRUE

3. If $\angle ABC$ and $\angle DEF$ are supplementary, then $\angle ABC$ and $\angle DEF$ form a linear pair.

false

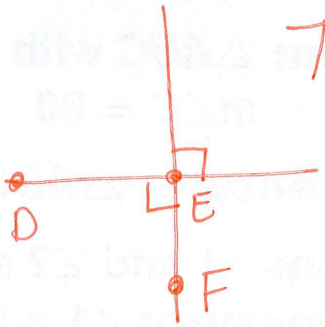


they are not a linear pair

or not adjacent

4. If $\overline{DE} \perp \overline{EF}$, then $\angle DEF$ is a right angle.

TRUE



Exit Tickets for lesson 2.1

Algebra 1 Exit ticket #1 Orders of Operations and Multiplying polynomials

Find each product and simplify if possible

1. $(4x^2 - 3x + 77)(5x - 3)$

$20x^3 - 12x^2 - 15x^2 + 9x + 385x - 231$

$20x^3 - 27x^2 + 394x - 231$

2. $3x^2 - 7xy + 9x(2x - 5y) + 3y^2$

$3x^2 - 7xy + 18x^2 - 45xy + 3y^2$

$21x^2 - 52xy + 3y^2$

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

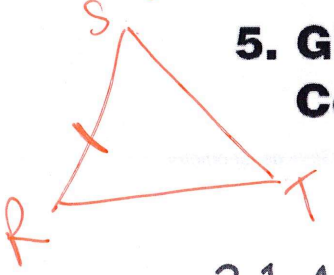
3. Given: $\triangle ABC$ with $m\angle A = 60$, $m\angle B = 60$, and $m\angle C = 60$ TRUE

Conjecture: $\triangle ABC$ is equilateral.

4. Given: $\angle 1$ and $\angle 2$ are supplementary angles.
Conjecture: $\angle 1$ and $\angle 2$ are congruent. false

$m\angle 1 = 70$ $m\angle 2 = 110$

5. Given: $\triangle RST$ has two congruent sides.
Conjecture: $\overline{RS} \cong \overline{ST}$ false because it could be $RS \cong RT$



2.1 Assign: pg. 94 55,57,65,66,68

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2-2 Study Guide and Intervention

Logic

Determine Truth Values A **statement** is any sentence that is either true or false.
The **truth value** of a statement is either true (T) or false (F). A statement can be represented by using a letter.
For example, Statement *p*: Chicago is a city in Illinois. The truth value of statement *p* is true.
Several statements can be joined in a Compound Statement

Negation: not *p* is the negation of the statement *p*.	Statement *p* and statement *q* joined by the word and is a Conjunction	Statement *p* and statement *q* joined by the word OR is Disjunction
Symbols: ~p Read: not *p*)	Symbols: p∧q (Read: *p and q*)	Symbols: p∨q Read p or q
The statements *p* and ~*p* have opposite Truth Values	The conjunction *p* ∧ *q* is true only when both p & q are true	The disjunction *p* ∨ *q* is true if *p* is true, if q is true, or if both are true

Example 1: Write a compound statement for each conjunction. Then find its truth value.

p: An elephant is a mammal.

q: A square has four right angles.

a. $p \wedge q$
Join the statements with *and*: An elephant is a mammal and a square has four right angles. Both parts of the statement are true so the compound statement is true.

p∧q TRUE

b. $\sim p \wedge q$
~*p* is the statement "An elephant is not a mammal." Join ~*p* and *q* with the word *and*: An elephant is not a mammal and a square has four right angles. The first part of the compound statement, ~*p*, is false. Therefore the compound statement is false.

~p∧q ← false

Example 2: Write a compound statement for each disjunction. Then find its truth value.

p: A diameter of a circle is twice the radius.

q: A rectangle has four equal sides.

a. $p \vee q$ OR
Join the statements *p* and *q* with the word *or*: A diameter of a circle is twice the radius or a rectangle has four equal sides. The first part of the compound statement, *p*, is true, so the compound statement is true.

p is true so p∨q TRUE

b. $\sim p \vee q$
Join ~*p* and *q* with the word *or*: A diameter of a circle is not twice the radius or a rectangle has four equal sides. Neither part of the disjunction is true, so the compound statement is false

false because both false

Exercises

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

p: 10 + 8 = 18 *q*: September has 30 days. *r*: A rectangle has four sides.

1. *p* and *q* 10+8=18 and Sept has 30 days p∧q TRUE Both True p & q
2. $p \vee r$ 10+8=18 OR A rectangle has 4 sides p true TRUE
3. *q* or *r* Sept 30 days or rectangle has 4 sides TRUE
4. $q \wedge \sim r$ A rectangle has 4 sides and Rectangle does Not have 4 sides False because ~r is false both have to be true

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2-2 Study Guide and Intervention *(continued)*

Logic

Truth Tables One way to organize the truth values of statements is in a **truth table**. The truth tables for negation, conjunction, and disjunction are shown at the right.

Negation	
q	$\sim p$
T	F
F	T

AND

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

OR

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	F
F	T	F
F	F	F

Example 1: Construct a truth table for the compound statement q or r.
Use the disjunction table.

q	r	q or r
T	T	T
T	F	T
F	T	T
F	F	F

Example 2: Construct a truth table for the compound statement p and (q or r).
Use the disjunction table for (q or r). Then use the conjunction table for p and (q or r).

p	q	r	q or r	p and (q or r)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

Exercises
Construct a truth table for each compound statement.

1. p or r

p	r	p or r
T	T	T
T	F	T
F	T	T
F	F	F

only false when both false

2. $\sim p \vee q$ OR

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

3. $q \wedge \sim r$

4. $\sim p \wedge \sim r$

5. (p and r) or q

p	$\sim p$	r	$\sim r$	$\sim p \wedge \sim r$ (and)
T	F	T	F	F
T	F	F	T	F
F	T	T	F	F
F	T	F	T	T

32. CCSS **REASONING** Venus has switches at the top and bottom of her stairs to control the light for the stairwell. She notices that when the upstairs switch is up and the downstairs switch is down, the light is turned on.

Position of Switch		Light On
Upstairs	Downstairs	
up	up	F
up	down	T
down	up	T
down	down	F

a. Copy and complete the truth table.

b. If both the upstairs and downstairs switches are in the up position, will the light be on? Explain your reasoning.

No when both switches are up the value is false in the light on column

c. If the upstairs switch is in the down position and the downstairs switch is in the up position, will the light be on? yes

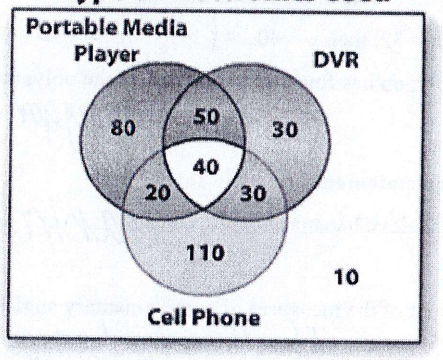
when the upstairs switch is down and the downstairs switch is up the value is true light is on

d. In general, how should the two switches be positioned so that the light is on?

the light is on when switches are opposite

33. ELECTRONICS A group of 330 teens were surveyed about what type of electronics they used. They chose from a cell phone, a portable media player, and a DVR. The results are shown in the Venn diagram.

Type of Electronics Used



a. How many teens used only a portable media player and DVR? 50

b. How many said they used all three types of electronics? 40

c. How many said they used only a cell phone? 110

d. How many teens said they used only a portable media player and a cell phone? 20

e. Describe the electronics that the 10 teens outside of the regions use.

they don't use any electronic device

Assign: Algebra Review wksh

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2-3 Study Guide and Intervention

Conditional Statements

If-Then Statements An if-then statement is a statement such as "If you are reading this page, then you are studying math." A statement that can be written in if-then form is called a Conditional Stater The phrase immediately following the word *if* is the **hypothesis**. The phrase immediately following the word *then* is the Conclusion

A conditional statement can be represented in symbols as $p \rightarrow q$ which is read "*p* implies *q*" or "if *p*, then *q*."

Example 1: Identify the hypothesis and conclusion of the conditional statement.

If $\underbrace{\angle X \cong \angle R \text{ and } \angle R \cong \angle S}_{\textit{hypothesis}}$, then $\underbrace{\angle X \cong \angle S.}_{\textit{conclusion}}$

Example 2: Identify the hypothesis and conclusion. Write the statement in if-then form.

You receive a free pizza with 12 coupons.

If You have 12 Coupons (*hypothesis*) then You get free Pizza (*conclusion*)

Exercises

Identify the hypothesis and conclusion of each conditional statement.

1. If it is Saturday, then there is no school. H: It is Saturday C. There is no school

2. If $x - 8 = 32$, then $x = 40$. H: $X-8=32$ C: $X=40$

3. If a polygon has four right angles, then the polygon is a rectangle.

H: a polygon has 4 right angles C: the polygon is a rectangle

Write each statement in if-then form.

4. All apes love bananas. If an animal is an ape, then it loves bananas

5. The sum of the measures of complementary angles is 90.

If 2 angles are complementary then the sum of measures is 90.

6. Collinear points lie on the same line.

If points are Collinear, then they lie on the same line.

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

7. If today is Wednesday, then yesterday was Friday.

False

If today was Wednesday then yesterday would be Tuesday always

C: So false

8. If *a* is positive, then 10*a* is greater than *a*.

TRUE

If a is positive then 10a is greater than a always

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2-3 Study Guide and Intervention *(continued)*

Conditional Statements

Converse, Inverse, and Contrapositive If you change the hypothesis or conclusion of a conditional statement, you form **related conditionals**. This chart shows the three related conditionals, *converse*, *inverse*, and *contrapositive*, and how they are related to a conditional statement.

	Symbols	Formed by	Example
Conditional	$p \rightarrow q$	using given hypothesis & Conclusi	If two angles are vertical angles, then they are congruent.
Converse	$q \rightarrow p$	exchanging the hypothesis and conclusion	If two angles are congruent, then they are vertical angles.
Inverse	$\sim p \rightarrow \sim q$	replacing hypothesis w/ negation and conclusion w/ negation	If two angles are not vertical angles, then they are not congruent.
Contrapositive	$\sim q \rightarrow \sim p$	negating the hypothesis, negating the conclusion, and switching them	If two angles are not congruent, then they are not vertical angles.

1 2

False

Not p then not q

Just as a conditional statement can be true or false, the related conditionals also can be true or false. A conditional statement always has the same truth value as its contrapositive, and the converse and inverse always have the same truth value.

Exercises

Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

1. If you live in San Diego, then you live in California.

If you live in Cali

look at notes

2. If a polygon is a rectangle, then it is a square.

3. If two angles are complementary, then the sum of their measures is 90.

Exit ticket: 2.3

Identify the hypothesis and conclusion of each conditional statement.

1. If $6x - 5 = 19$, then $x = 4$. Hyp: $6x-5=19$ Conc: $x=4$

2. A polygon is a hexagon if it has six sides.
h: A polygon has six sides C: then it is a hexagon

Write each statement in if-then form.

3. Exercise makes you healthier.
If you exercise then you will be healthier

4. Squares have 4 sides.
If a figure is a square, then it has 4 sides.

2-3 Assign: pg. 111 19-51 EOO 59-61 Extra 69-72

Quiz after this lesson

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2-4 Study Guide and Intervention
Deductive Reasoning

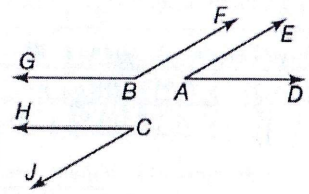
Law of Detachment **Deductive reasoning** is the process of using facts, rules, definitions, or properties to reach conclusions. One form of deductive reasoning that draws conclusions from a true conditional $p \rightarrow q$ and a true statement p is called the **Law of Detachment**.

Law of Detachment	If $p \rightarrow q$ is true and p is true then q is true

Example: Determine whether each conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

a. Given: Two angles supplementary to the same angle are congruent.
$\angle A$ and $\angle C$ are supplementary to $\angle B$.

Conclusion: $\angle A$ is congruent to $\angle C$.



The Statement $\angle A$; $\angle C$ are Supplementary to $\angle B$ is the hypothesis of the conditional by Law of Detachment the Conclusion is true

b. Given: If Helen is going to work, then she is wearing pearls. Helen is wearing pearls.

Conclusion: Helen is going to work.

The given Helen is going to work

Exercises

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

1. Given: If a number is divisible by 6, then the number is divisible by 3. 18 is divisible by 6.
Conclusion: 18 is divisible by 3. TRUE Valid law of Detachment

2. Given: If a pet is a rabbit, then it eats carrots. Jennie's pet eats carrots.
Conclusion: Jennie's pet is a rabbit. Invalid Jennies Pet Could be dog

3. Given: If a hen is a Plymouth Rock, then her eggs are brown. Berta is a Plymouth Rock hen.
Conclusion: Berta's eggs are brown.

Valid the hypothesis of the true Conditional is true so Conclusion is true

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2-4 Study Guide and Intervention *(continued)*

Deductive Reasoning

Law of Syllogism Another way to make a valid conclusion is to use the **Law of Syllogism**. It allows you to draw conclusions from two true statements when the conclusion of one statement is the hypothesis of another.

Law of Syllogism	If $p \to q$ is true and $q \to r$ true then $p \to r$ true

Example: The two conditional statements below are true. Use the Law of Syllogism to find a valid conclusion. State the conclusion.

(1) If a number is a whole number, then the number is an integer.
(2) If a number is an integer, then it is a rational number.

p: Number is whole #
q: the # is an integer
r: a # is a rational #

$p \to q$ $q \to r$ $\Rightarrow$

The two conditional statements are $p \to q$ and $q \to r$. Using the Law of Syllogism, a valid conclusion is $p \to r$.
A statement of $p \to r$ is If a # is a whole #, then it is rational #

Exercises
Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion is possible, write *no valid conclusion*.

1. If a dog eats Superdog Dog Food, he will be happy. Rover is happy.

Invalid no Valid Conclusion

2. If an angle is supplementary to an obtuse angle, then it is acute. (p, q)
If an angle is acute, then its measure is less than 90. (r)

$p \to q$ $q \to r$ So $p \to r$ (Valid)

3. If the measure of $\angle A$ is less than 90, then $\angle A$ is acute. If $\angle A$ is acute, then $\angle A \cong \angle B$.
(p) (q) (r)

4. If an angle is a right angle, then the measure of the angle is 90.
If two lines are perpendicular, then they form a right angle.

5. If you study for the test, then you will receive a high grade.
Your grade on the test is high.

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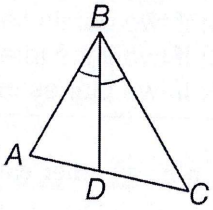
2-5 Study Guide and Intervention *(continued)*

Postulates and Paragraph Proofs

Paragraph Proofs A logical argument that uses deductive reasoning to reach a valid conclusion is called a [ ]. In one type of proof, a [ ] you write a paragraph to explain why a statement is true.

A statement that can be proved true is called a [ ]. You can use undefined terms, definitions, postulates, and already-proved theorems to prove other statements true.

Example: In $\triangle ABC$, $\overline{BD}$ is an angle bisector. Write a paragraph proof to show that $\angle ABD \cong \angle CBD$.



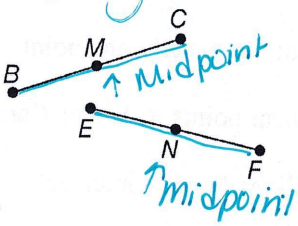
By definition, an angle bisector divides an angle into two congruent angles, Since $\overline{BD}$ is an angle bisector $\angle ABC$ is divided into two congruent angles. Therefore $\angle ABD \cong \angle CBD$

Exercises

1. Given that $\angle A \cong \angle D$ and $\angle D \cong \angle E$, write a paragraph proof to show that $\angle A \cong \angle E$.

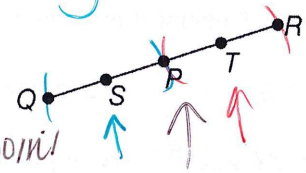
Since $\angle A \cong \angle D$ and $\angle D \cong \angle E$ the $\angle A \cong \angle E$ then, $m\angle A = m\angle D$ and $m\angle D = m\angle E$ by def of congruence. So $m\angle A = m\angle E$ by Transitive property and $\angle A \cong \angle E$ by def of congruence

2. It is given that $\overline{BC} \cong \overline{EF}$, M is the midpoint of $\overline{BC}$, and N is the midpoint of $\overline{EF}$. Write a paragraph proof to show that $BM = EN$.



M is the midpoint of $\overline{BC}$ and N is the midpoint of $\overline{EF}$. So $BM = \frac{1}{2}BC$ and $EN = \frac{1}{2}EF$. $\overline{BC} \cong \overline{EF}$ so $BC = EF$ by def of congruence and $\frac{1}{2}BC = \frac{1}{2}EF$ by Mult Prop thus $BM = EN$

3. Given that S is the midpoint of $\overline{QP}$, T is the midpoint of $\overline{PR}$, and P is the midpoint of $\overline{ST}$, write a paragraph proof to show that $QS = TR$.



S is the Midpoint of $\overline{QP}$ so $QS = SP$. P is the Midpoint of $\overline{ST}$ so $SP = PT$. Thus $QS = PT$ by transitive property. T is the Midpoint of PR, so $\overline{PT} = PR$ ∴ $QS = PR$ by transitive Property

① S is Midpoint $\overline{QP}$ P is Midpoint $\overline{ST}$ T is the Midpoint of $\overline{PR}$	① Given
② $QS = SP$; $SP = PT$; $PT = Q$	② Def Midpoint
③ $QS = PT$	③ transitive pr
④ $QS = PR$	transitive

① M is Midpoint $\overline{BC}$ N is Midpoint $\overline{EF}$	① Given
② $BM = \frac{1}{2}BC$	Def Midpoint
$EN = \frac{1}{2}EF$	
③ $\overline{BC} \cong \overline{EF}$ so $BC = EF$	Def of Congruence
④ $\frac{1}{2}BC = \frac{1}{2}EF$	Mult Prop
⑤ $BC = EN$	Substitution transitive Prop

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2-5 Study Guide and Intervention

Postulates and Paragraph Proofs

Points, Lines, and Planes In geometry, a **postulate** is a statement that is accepted as true. Postulates describe fundamental relationships in geometry.

Postulate 2.1: Through any two points, there is exactly one line.
Postulate 2.2: Through any three noncollinear points, there is exactly one plane.
Postulate 2.3: A line contains at least two points.
Postulate 2.4: A plane contains at least three noncollinear points.
Postulate 2.5: If two points lie in a plane, then the entire line containing those points lies in the plane.
Postulate 2.6: If two lines intersect, then their intersection is exactly one point.
Postulate 2.7: If two planes intersect, then their intersection is a line.

Example: Determine whether each statement is *always, sometimes, or never true.*

a. There is exactly one plane that contains points *A*, *B*, and *C*.

b. Points *E* and *F* are contained in exactly one line.

c. Two lines intersect in two distinct points *M* and *N*.

Exercises

Determine whether each statement is *always, sometimes*, or *never* true.

1. A line contains exactly one point.

2. Noncollinear points R, S, and T are contained in exactly one plane.

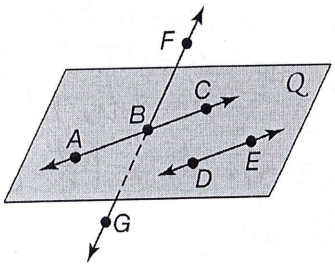
3. Any two lines ℓ and m intersect.

4. If points G and H are contained in plane $\mathcal{M}$, then $\overline{GH}$ is perpendicular to plane $\mathcal{M}$.

5. Planes $\mathcal{R}$ and $\mathcal{S}$ intersect in point T.

6. If points A, B, and C are noncollinear, then segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are contained in exactly one plane.

In the figure, $\overline{AC}$ and $\overline{DE}$ are in plane Q and $\overline{AC} \parallel \overline{DE}$. State the postulate that can be used to show each statement is true.



7. Exactly one plane contains points F, B, and E.

8. $\overleftrightarrow{BE}$ lies in plane Q.

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2-6 Study Guide and Intervention

Algebraic Proof

Algebraic Proof A list of algebraic steps to solve problems where each step is justified is called an **algebraic proof**. The table shows properties you have studied in algebra.

The following properties are true for any real numbers *a*, *b*, and *c*.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a=b$ then $a-c=b-c$
Mult Property of Equality	If $a = b$, $a \cdot c = b \cdot c$
Division Property of Equality	If $a=b$ $\frac{a}{c}=\frac{b}{c}$ $c \neq 0$
Reflexive Property of Equality	$a=a$
Symmetric Property of Equality	$a=b$ then $b=a$
Transitive Property of Equality	If $a=b$ the $b=c$ then $a=c$
Substitution of Equality	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive prop	$a(b + c) = ab + ac$

Whatever you do to one side you do to the other

Example: Solve $6x + 2(x - 1) = 30$. Write a justification for each step.

Algebraic Steps	Properties
$6x + 2(x - 1) = 30$	Original equation or Given
$10x+2x-2=30$	Distributive Property
$8x-2=30$	Substitution Property of Equality
$8x-2+2=30+2$	Addition Property of Equality
$8x=32$	Substitution Property of Equality
$\frac{8x}{8}=\frac{32}{8}$	Division Property of Equality
$x=4$	Substitution Property of Equality

You know this

Exercises

Complete each proof.

1. Given: $\frac{4x+6}{2} = 9$

Prove: $x = 3$

Proof:

Statements	Reasons
a. $\frac{4x+6}{2} = 9$	a. Given
b. $2\left(\frac{4x+6}{2}\right) = 2(9)$	b. Mult. Prop.
c. $4x + 6 = 18$	c. Substitution
d. $4x + 6 - 6 = 18 - 6$	d. Subtraction prop
e. $4x =$ 12	e. Substitution
f. $\frac{4x}{4} =$ $\frac{12}{4}$	f. Div. Prop.
g. $x=3$	g. Substitution

2. Given: $4x + 8 = x + 2$

Prove: $x = -2$

Proof:

Statements	Reasons
a. $4x + 8 = x + 2$	a. Given
b. $4x + 8 - x = x + 2 - x$	b. Subtraction prop
c. $3x + 8 = 2$ $-8 \quad -8$	c. Substitution do the Math
d. $3x+8-8=2-8$	d. Subtr. Prop.
e. $3x=-6$	e. Substitution
f. $\frac{3x}{3} = \frac{-6}{3}$	f. Division prop
g. $x=-2$	g. Substitution

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2-6 Study Guide and Intervention *(continued)*

Algebraic Proof

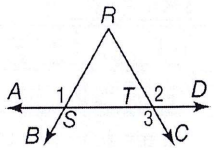
Geometric Proof Geometry deals with numbers as measures, so geometric proofs use properties of numbers. Here are some of the algebraic properties used in proofs.

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$ then $CD = AB$	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$

Example: Write a two-column proof to verify this conjecture.

Given: $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$

Prove: $m\angle 1 = m\angle 3$



Proof:

Statements	Reasons
1. $m\angle 1 = m\angle 2$	1. Given
2. $m\angle 2 = m\angle 3$	2. Given
3. $m\angle 1 = m\angle 3$	3. Transitive Property of Equality

Exercises

State the property that justifies each statement.

1. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$. Symmetric
2. If $m\angle 1 = 90$ and $m\angle 2 = m\angle 1$, then $m\angle 2 = 90$. $m\angle 1 = 90$ $m\angle 2 = m\angle 1$ S
   Substitution
3. If $AB = RS$ and $RS = WY$, then $AB = WY$.
   Transitive
4. If $AB = CD$, then $\frac{1}{2}AB = \frac{1}{2}CD$. I can Mult both sides by $\frac{1}{2}$ if start =
   Mult prop
5. If $m\angle 1 + m\angle 2 = 110$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 110$.
   Substitution
6. $RS = RS$ Reflexive
7. If $AB = RS$ and $TU = WY$, then $AB + TU = RS + WY$. Addition prop
8. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$. Transitive
9. If the formula for the area of a triangle is $A = \frac{1}{2}bh$, then bh is equal to 2 times the area of the triangle. Write a two-column proof to verify this conjecture. $A = \frac{1}{2}bh$ Prove $2A = bh$

Statements	Reasons
1 $A = \frac{1}{2}bh$	1 Given
2 $2A = \frac{1}{2}bh \cdot 2$	2 Mult prop
3 $A = bh$	3 Substitution

Exit ticket 2.6

State the property that justifies each statement.

1. $2(LM + NO) = 2LM + 2NO$ Dist

2. If $m\angle R = m\angle S$, then $m\angle R + m\angle T = m\angle S + m\angle T$. Add

3. If $2PQ = OQ$, then $PQ = \frac{1}{2}OQ$. Division

4. $m\angle Z = m\angle Z$ Ref

5. If $BC = CD$ and $CD = EF$, then $BC = EF$. Transitive

Standardized Test Practice

6. Which statement shows an example of the Symmetric Property?

A $x = x$

B If $x = 3$, then $x + 4 = 7$.

C If $x = 3$, then $3 = x$.

D If $x = 3$ and $x = y$, then $y = 3$.

2.6 Assign:pg. 141 23-26 you pick 2

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2-7 Study Guide and Intervention

Proving Segment Relationships

Segment Addition Two basic postulates for working with segments and lengths are the Ruler Postulate, which establishes number lines, and the Segment Addition Postulate, which describes what it means for one point to be between two other points.

Ruler Postulate	The points on any line or line segment can be put into one-to-one with real numbers.
Segment Addition Postulate	If A, B, and C are collinear, then point B is between A and C if and only if $AB + BC = AC$.

Example: Write a two-column proof.

Given: Q is the midpoint of $\overline{PR}$.
R is the midpoint of $\overline{QS}$.

Prove: $PR = QS$

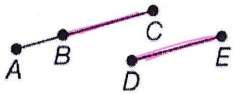
Proof:

Statements	Reasons
1. Q is the midpoint of $\overline{PR}$.	1. Given
2. PQ = QR	2. Definition of midpoint
3. R is the midpoint of $\overline{QS}$.	3. Given
4. QR = RS	4. Definition of midpoint
5. $PQ = RS$	5 Transitive prop
6. PQ + QR = RS + QR	6. Addition Property
7. $PQ + QR = PR$, $QR + RS = QS$	7. Segment Addition Pos
8 PR = QS	8. Substitution

Exercises

Complete each proof.

1. **Given:** $BC = DE$
Prove: $AB + DE = AC$



Proof:

Statements	Reasons
1. $BC = DE$	1. Given
2. BC + AB = AC	2. Seg. Add. Post.
3. $AB + DE = AC$	3. Substitution

2. **Given:** Q is between P and R, R is between Q and S, $PR = QS$.
Prove: $PQ = RS$

Proof:

Statements	Reasons
1. Q is between P and R.	1. Given
2. $PQ + QR = PR$	2. Seg Add Post
3. R is between Q and S.	3. Given
4. QR + RS = QS	4. Seg. Add. Post.
5. $PR = QS$	5. Given
6. $PQ + QR = QR + RS$	6. Substitution
7. $PQ + \cancel{QR} - \cancel{QR} = \cancel{QR} + RS - \cancel{QR}$	7. Subtraction prop
8. PQ = RS	8. Substitution

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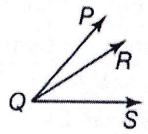
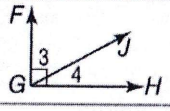
2-8 Study Guide and Intervention
Proving Angle Relationships

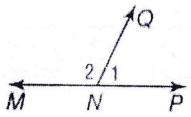
Supplementary and Complementary Angles There are two basic postulates for working with angles. The Protractor Postulate assigns numbers to angle measures, and the Angle Addition Postulate relates parts of an angle to the whole angle.

Protractor Postulate	Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180	
Angle Addition Postulate	R is in the interior of $\angle PQS$ if and only if m∠PQS; m∠PQR + m∠RQS =	

The two postulates can be used to prove the following two theorems.

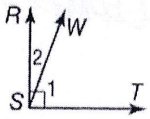
Supplement Theorem	If two angles form a linear pair, then they are Supplementary **Example:** If $\angle 1$ and $\angle 2$ form a linear pair, then $m\angle 1 + m\angle 2 = 180$	
Complement Theorem	If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. **Example:** If GF ⊥ GH the m∠3 + m∠4 = 90	

Example 1: If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 115$, find $m\angle 1$.



$m\angle 1 + m\angle 2 = 180$ Supplement thm
$m\angle 1 + 115 = 180$ Substitution
$m\angle 1 = 65$ Subtraction

Example 2: If $\angle 1$ and $\angle 2$ form a right angle and $m\angle 2 = 20$, find $m\angle 1$.

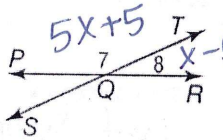


m∠1 + m∠2 = 90 Compl. Theorem
m∠1 + 20 = 90 Substitution
m∠1 = 70° Subtraction Prop.

Exercises

Find the measure of each numbered angle and name the theorem that justifies your work.

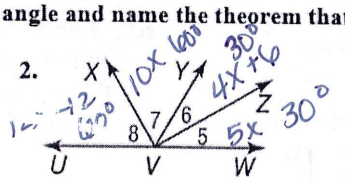
1.



$m\angle 7 = 5x + 5$,
$m\angle 8 = x - 5$

$5x+5 + x-5 = 180$
$\frac{6x}{6} = \frac{180}{6}$
$x = 30$
$m\angle 8 = 155°$
$m\angle 7 = 25°$

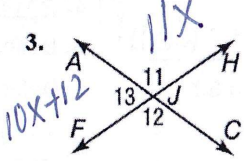
2.



$m\angle 5 = 5x$, $m\angle 6 = 4x + 6$,
$m\angle 7 = 10x$,
$m\angle 8 = 12x - 12$

$12x-12 + 10x + 4x+6 + 5x = 180$
$31x - 6 = 180$
$31x = 186$
$x = 6$
$m\angle 5 = 30°$
$m\angle 6 = 30°$
$m\angle 7 = 60°$
$m\angle 8 = 60°$

3.



$m\angle 11 = 11x$,
$m\angle 13 = 10x + 12$

$11x + 10x + 12 = 180$
$21x + 12 = 180$
$-12 \quad -12$
$21x = 168$
$x = 8$
$m\angle 11 = 88°$
$m\angle 12 = 88°$
$m\angle 13 = 92°$

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2-8 Study Guide and Intervention *(continued)*

Proving Angle Relationships

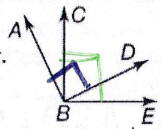
Congruent and Right Angles The Reflexive Property of Congruence, Symmetric Property of Congruence, and Transitive Property of Congruence all hold true for angles. The following theorems also hold true for angles.

Congruent Supplements Theorem	Angles supplement to the same angle or congruent angles are congruent.
Congruent Complements Theorem	Angles Complement to the same angle or to Congruent angles are Congruent
Vertical Angles Theorem	If 2 angles are Vertical they are Congruent
Theorem 2.9	Perpendicular lines intersect to form 4 right angles
Theorem 2.10	All Right ∠'s are congruent.
Theorem 2.11	Perpendicular lines form Congruent adjacent angles
Theorem 2.12	If 2 angles are Cong & Supplement then each angle is a right angle.
Theorem 2.13	If two congruent angles form a linear pair then they are right angles

Example: Write a two-column proof.

Given: $\angle ABC$ and $\angle CBD$ are complementary.
$\angle DBE$ and $\angle CBD$ form a right angle.

Prove: $\angle ABC \cong \angle DBE$

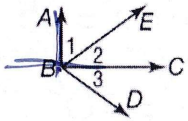


Statements	Reasons
1. $\angle ABC$ and $\angle CBD$ are complementary. $\angle DBE$ and $\angle CBD$ form a right angle.	1. Given
2. ∠DBE and ∠CBD are Complem	2. Complement Theorem
3. $\angle ABC \cong \angle DBE$	3. ∠'s Comp to Same ∠ or ≅∠'s are ≅

Exercises

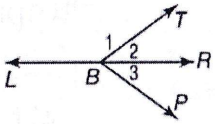
Complete each proof.

1. Given: $\overline{AB} \perp \overline{BC}$; $\angle 1$ and $\angle 3$ are complementary.
Prove: $\angle 2 \cong \angle 3$
Proof:



Statements	Reasons
a. $\overline{AB} \perp \overline{BC}$	a. Given
b. ∠ABC is Right	b. Definition of ⊥
c. $m\angle ABC = 90$	c. Def. of right angle
d. $m\angle ABC = m\angle 1 + m\angle 2$	d. ∠ Add Post
e. $90 = m\angle 1 + m\angle 2$	e. Substitution
f. $\angle 1$ and $\angle 2$ are compl.	f. Def Comp ∠'s
g. ∠1 an ∠3 Comp	g. Given
h. $\angle 2 \cong \angle 3$	h. ∠'s Comp to the Sam ∠ is ≅

2. Given: $\angle 1$ and $\angle 2$ form a linear pair.
$m\angle 1 + m\angle 3 = 180$
Prove: $\angle 2 \cong \angle 3$
Proof:



Statements	Reasons
a. $\angle 1$ and $\angle 2$ form a linear pair. $m\angle 1 + m\angle 3 = 180$	a. Given
b. ∠1 & ∠2 Supp	b. Suppl. Theorem
c. $\angle 1$ is suppl. to $\angle 3$.	c. Def Supp
d. ∠2 ≅ ∠3	d. ∠s suppl. to the same ∠ or ≅∠s are ≅.