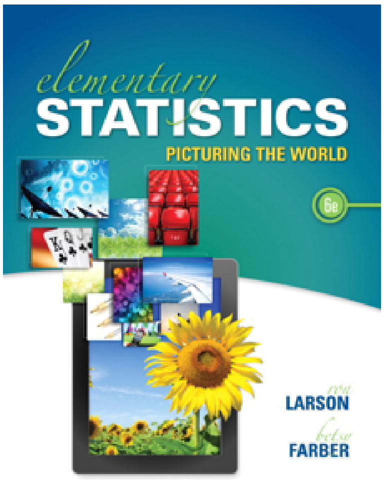
Elementary Statistics: Picturing The World

Sixth Edition



Chapter 7

Hypothesis Testing with One Sample

7.1 pg. 359 11-16,21,22,23,26,29,30,37,41,42,44,47,49,50

7.2 pg. 373 3,5,7,10,12,14,16,18,19,21,24,25,27,30,31,39

7.3 pg. 383 3,5,7,11,12,14,15,21,22,23,27

7.4 pg. 391 3,8,9-11,15 OR 16,17

7.5 pg. 400 5,7,9,12,13,15,18,19,21,23

Chapter 7: Hypothesis Testing with One Sample

7-1: Introduction to Hypothesis Testing

In a nutshell: Hypothesis testing looks at if a statement seems likely to be false or not.

For instance: An auto company may claim their new cars get 50 mpg. You wonder if this is really true. You can't test all the new cars though, so now what?

- First you take a sample of 30 cars. You find the mean mpg is 47 with a standard deviation of 5.5. This is NOT exactly 50, but we wouldn't expect it to be. How different is it though from their claim?

- WE ASSUME THEY ARE CORRECT. Then we look at what would be weird values (outside of 2 standard deviations) from their claim about the mean being 50.

If 47 turns out to be more than 2s.d. away, we would be very suspicious about their claim.

If 47 was closer than 2 s.d. from the mean of 50, we would have to assume they might be right.

Since we assume they are truthful, the null hypothesis always includes an equality. The other option, that they are wrong, contains no equality and is the complement of their claim (the opposite of their statement).

DEFINITION

1. A **null hypothesis** H_0 is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$, or $\geq$.
2. The **alternative hypothesis** H_a is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and it contains a statement of strict inequality, such as $>$, $\neq$, or $<$.

H_0 is read as "H subzero" or "H naught" and H_a is read as "H sub-a."

We can write null and alternatives for any parameter - μ p σ, σ^2

7-1: Continued

Verbal Statement H_0 *The mean is . . .*	Mathematical Statements	Verbal Statement H_a *The mean is . . .*
. . . greater than or equal to k. *. . . at least k.* *. . . not less than k.*	$\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$	*. . . less than k.* *. . . below k.* *. . . fewer than k.*
. . . less than or equal to k. *. . . at most k.* *. . . not more than k.*	$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$	*. . . greater than k.* *. . . above k.* *. . . more than k.*
. . . equal to k. *. . . k.* *. . . exactly k.*	$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$	*. . . not equal to k.* *. . . different from k.* *. . . not k.*

$$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases} \qquad \begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases} \qquad \begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$$

Example 1: Stating Null/Alternative Hypotheses

Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. A car dealership announces that the mean time for an oil change is less than 15 minutes.
3. A company advertises that the mean life of its furnaces is more than 18 years.

EXAMPLE 2

Identifying Type I and Type II Errors

The USDA limit for salmonella contamination for chicken is 20%. A meat inspector reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which error is more serious?

TYPES OF ERRORS AND LEVEL OF SIGNIFICANCE

No matter which hypothesis represents the claim, you always begin a hypothesis test by assuming that the equality condition in the null hypothesis is true. So, when you perform a hypothesis test, you make one of two decisions:

1. reject the null hypothesis

or

2. fail to reject the null hypothesis.

DEFINITION

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

The following table shows the four possible outcomes of a hypothesis test.

	Truth of H_0	
Decision	H_0 is true.	H_0 is false.
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

7-1 continued... Errors, levels of significance

Types of errors:

Level of significance: All samples will vary some from the population parameter (remember our errors from CIs?). How often will you reject the null hypothesis when it is actually true (false positive)? 5% of the time? 1%? 10%? This is your LEVEL OF SIGNIFICANCE.

DEFINITION

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a type I error. It is denoted by α, the lowercase Greek letter alpha.

	Truth About Defendant	
Verdict	**Innocent**	**Guilty**
Not guilty	Justice	Type II error
Guilty	Type I error	Justice

1. A carefully worded accusation is written.
2. The defendant is assumed innocent (H_0) until proven guilty. The burden of proof lies with the prosecution. If the evidence is not strong enough, then there is no conviction. A "not guilty" verdict does not prove that a defendant is innocent.
3. The evidence needs to be conclusive beyond a reasonable doubt. The system assumes that more harm is done by convicting the innocent (type I error) than by not convicting the guilty (type II error).

What we want is to be able to compare our test statistic's value versus the value that is claimed to be true, which means we need to convert them into our standard distributions (z-scores, t-scores, or chi-squared scores).

Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 7.2, σ known), t (Section 7.3, σ unknown)
p	$\hat{p}$	z (Section 7.4)
σ^2	s^2	χ^2 (Section 7.5)

We then compare the P-value (probability) of getting a sample value as extreme or more extreme than the unusual ranges of our claim's distribution.

DEFINITION

If the null hypothesis is true, a ***P*-value** (or **probability value**) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

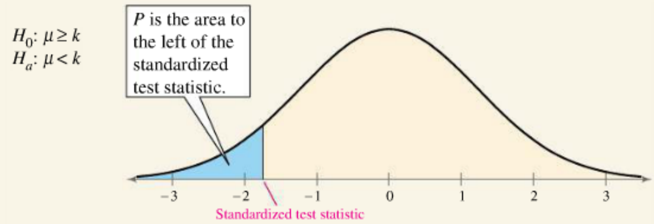
Reminder: (Summary of all of Chapter 6!)

7-1 continued... Type of test, P-values

The P-value of a hypothesis test depends on the nature of the test. There are three types of hypothesis tests—**left-tailed, right-tailed,** and **two-tailed.** The type of test depends on the location of the region of the sampling distribution that favors a rejection of H_0. This region is indicated by the alternative hypothesis.

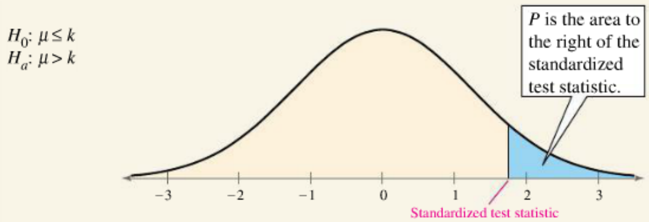
DEFINITION

1. If the alternative hypothesis H_a contains the less-than inequality symbol ($<$), then the hypothesis test is a **left-tailed test.**



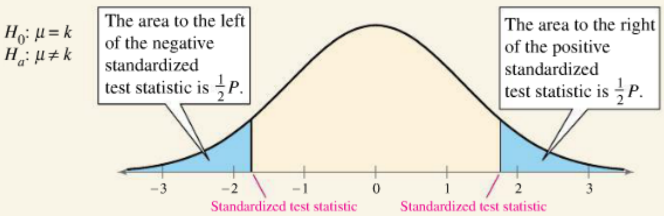
Left-Tailed Test

2. If the alternative hypothesis H_a contains the greater-than inequality symbol ($>$), then the hypothesis test is a **right-tailed test.**



Right-Tailed Test

3. If the alternative hypothesis H_a contains the not-equal-to symbol ($\neq$), then the hypothesis test is a **two-tailed test.** In a two-tailed test, each tail has an area of $\frac{1}{2}P$.



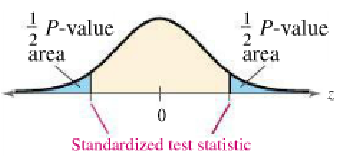
Two-Tailed Test

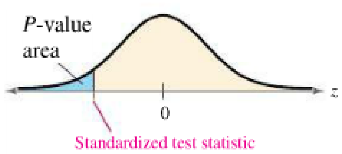
Example 3: Type of Hypothesis Test?

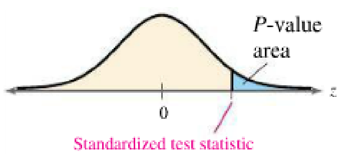
Identifying the Nature of a Hypothesis Test

For each claim, state H_0 and H_a in words and in symbols. Then determine whether the hypothesis test is a left-tailed test, right-tailed test, or two-tailed test. Sketch a normal sampling distribution and shade the area for the P-value.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. A car dealership announces that the mean time for an oil change is less than 15 minutes.
3. A company advertises that the mean life of its furnaces is more than 18 years.







7.1 Continued

DECISION RULE BASED ON P-VALUE

To use a P-value to make a conclusion in a hypothesis test, compare the P-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

The smaller the P-value of the test, the more evidence there is to reject the null hypothesis. A very small P-value indicates an unusual event. Remember, however, that even a very low P-value does not constitute proof that the null hypothesis is false, only that it is probably false.

	Claim	
Decision	**Claim is H_0.**	**Claim is H_a.**
Reject H_0.	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

You can't prove things true - only can say that evidence indicates that it is false, or that there is not enough evidence to say that it is false!

EXAMPLE 4

Interpreting a Decision

You perform a hypothesis test for each claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

1. H_0 (Claim): A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. H_a (Claim): A car dealership announces that the mean time for an oil change is less than 15 minutes.

7-1 continued... Strategies

Writing hypotheses depends on which side of the argument you believe. You want to be able to reject the null, so your belief should be in the alternative hypothesis.

Example 5: Writing the hypotheses

A medical research team is investigating the benefits of a new surgical treatment. One of the claims is that the mean recovery time for patients after the new treatment is less than 96 hours.

1. Write the hypotheses as if you are on the research team and want to support the claim.

2. Write the hypotheses as if you are on an opposing team and want to reject the claim.

GENERAL STEPS - USE THROUGHOUT THIS CHAPTER!!!

STEPS FOR HYPOTHESIS TESTING

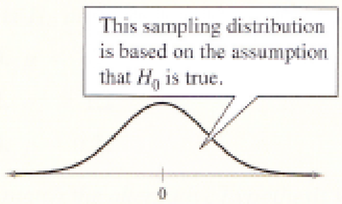
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

   H_0: ? H_a: ?

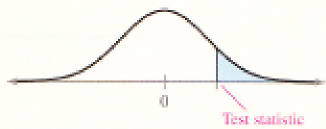
2. Specify the level of significance.

   $\alpha =$?

3. Determine the standardized sampling distribution and sketch its graph.



4. Calculate the test statistic and its standardized value. Add it to your sketch.



5. Find the P-value.
6. Use the following decision rule.

   Is the P-value less than or equal to the level of significance?
   - No → Fail to reject H_0.
   - Yes → Reject H_0.

7. Write a statement to interpret the decision in the context of the original claim.

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Chapter 7: Hypothesis Testing with One Sample

7-2: Hypothesis Tests when you know σ

One parameter we often want to do a hypothesis t ; the mean. Just like CIs, there are different methods if the sample is over 30 or under 30.

Reminder from yesterday:

DECISION RULE BASED ON *P*-VALUE

To use a P-value to make a conclusion in a hypothesis test, compare the P-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

Example 1: Interpreting a P-value

The P-value for a hypothesis test is P=0.0237. What is your decision if the level of significance is...

1) $\alpha = 0.05$

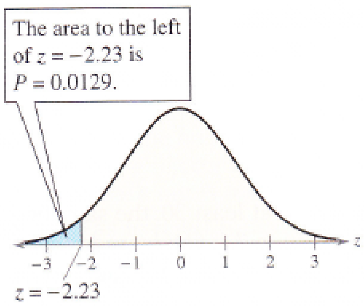
2) $\alpha = 0.01$

Finding those P-values is going to be very important, clearly! Use the tables (or for normal distributions, normalcdf(_, _))

FINDING THE *P*-VALUE FOR A HYPOTHESIS TEST

After determining the hypothesis test's standardized test statistic and the standardized test statistic's corresponding area, do one of the following to find the P-value.

a. For a left-tailed test, $P =$ (Area in left tail).
b. For a right-tailed test, $P =$ (Area in right tail).
c. For a two-tailed test, $P = 2$(Area in tail of standardized test statistic).



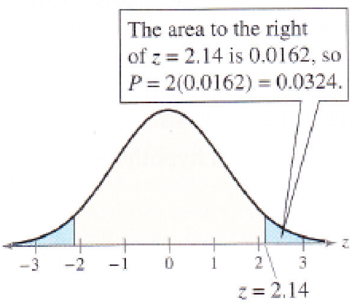
Left-Tailed Test

Example 2: Finding P-value for left-tailed tests

Find the P-value for a left-tailed hypothesis test with a test statistic of z=-2.23. Decide whether to reject the null if the level of significance is 0.01.

Example 2: Finding P-value for two-tailed tests

Find the P-value for a two-tailed hypothesis test with a test statistic of z=2.14. Decide whether to reject the null if the level of significance is 0.05.



Two-Tailed Test

7-2 continued... Using P-values for a z-test

If we know population Standard deviation , remember from chapter 6-1 that we always use the normal distribution.

z-TEST FOR A MEAN μ

The **z-test for a mean μ** is a statistical test for a population mean. The **test statistic** is the sample mean $\bar{x}$. The **standardized test statistic** is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$ Standardized test statistic for μ (σ known)

when these conditions are met.

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or $n \geq 30$.

Recall that $\sigma / \sqrt{n}$ is the standard error of the mean, $\sigma_{\bar{x}}$.

GUIDELINES

Using P-Values for a z-Test for a Mean μ (σ Known)

IN WORDS	IN SYMBOLS
1. Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Find the standardized test statistic.	$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
5. Find the area that corresponds to z.	Use Table 4 in Appendix B.

6. Find the P-value.
   a. For a left-tailed test, $P =$ (Area in left tail).
   b. For a right-tailed test, $P =$ (Area in right tail).
   c. For a two-tailed test, $P = 2$(Area in tail of standardized test statistic).

7. Make a decision to reject or fail to reject the null hypothesis.	If $P \leq \alpha$, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

7-2 Continued

Example 4: Hypothesis Testing for large samples

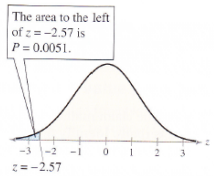
In an advertisement, a pizza shop claims that its mean delivery time is less than 30 minutes. A random selection of 36 delivery times has a sample mean of 28.5 minutes and a standard deviation of 3.5 minutes. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a P-value.

Solution:

1. State null and alt hypotheses.

2. Calculate the standardized score for the test statistic.

3. Decide if it is left, right, or two-tailed. Find the area that goes with the standardized score.



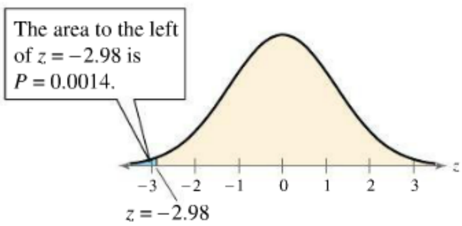
Left-Tailed Test

4. Decide if you should reject or fail to reject the null. Interpret the result.

EXAMPLE 4

Hypothesis Testing Using a *P*-Value

In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random sample of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a *P*-value.



Left-Tailed Test

7-2 continued... Using P-values for a z-test

Example 5: Hypothesis Testing for large samples

EXAMPLE 5

See Minitab steps on page 414.

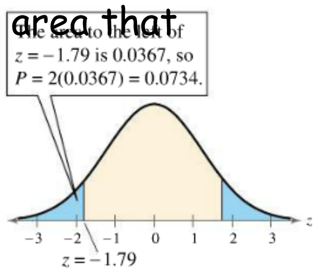
Hypothesis Testing Using a *P*-Value

According to a study, the mean cost of bariatric (weight loss) surgery is \$21,500. You think this information is incorrect. You randomly select 25 bariatric surgery patients and find that the mean cost for their surgeries is \$20,695. From past studies, the population standard deviation is known to be \$2250 and the population is normally distributed. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a *P*-value.

1. State null and alt hypotheses.

2. Calculate the standardized score for the test statistic

3. Decide if it is left, right, or two-tailed. Find the area that goes with the standardized score.



Two-Tailed Test

4. Decide if you should reject or fail to reject the null. Interpret the result.

EXAMPLE 6

Using Technology to Find a *P*-Value

Use the TI-84 Plus displays to make a decision to reject or fail to reject the null hypothesis at a level of significance of $\alpha = 0.05$.

Study Tip

Using a TI-84 Plus, you can either enter the original data into a list to find a *P*-value or enter the descriptive statistics.

STAT

Choose the TESTS menu.

1: Z-Test...

Select the *Data* input option when you use the original data. Select the *Stats* input option when you use the descriptive statistics. In each case, enter the appropriate values including the corresponding type of hypothesis test indicated by the alternative hypothesis. Then select *Calculate*.

7.2 Continued

DEFINITION

A **rejection region** (or **critical region**) of the sampling distribution is the range of values for which the null hypothesis is not probable. If a standardized test statistic falls in this region, then the null hypothesis is rejected. A **critical value** z_0 separates the rejection region from the nonrejection region.

GUIDELINES

Finding Critical Values in the Standard Normal Distribution

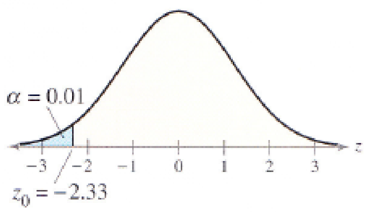
1. Specify the level of significance α.
2. Determine whether the test is left-tailed, right-tailed, or two-tailed.
3. Find the critical value(s) z_0. When the hypothesis test is
   a. *left-tailed,* find the z-score that corresponds to an area of α.
   b. *right-tailed,* find the z-score that corresponds to an area of $1 - \alpha$.
   c. *two-tailed,* find the z-scores that correspond to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution. Draw a vertical line at each critical value and shade the rejection region(s). (See the figures at the left.)

Note that a standardized test statistic that falls in a rejection region is considered an unusual event.

HINT: Draw a picture
USE INVNORM!

Example 7: Finding Critical Values for Right-tailed Test

Find a critical value and rejection region for a right-tailed test with $\alpha = 0.01$

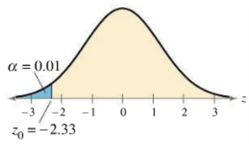


1% Level of Significance

EXAMPLE 7

Finding a Critical Value for a Left-Tailed Test

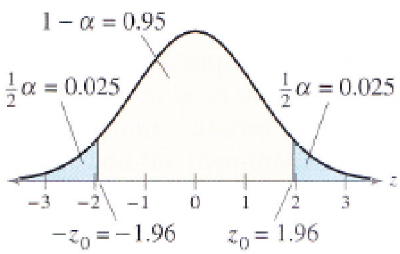
Find the critical value and rejection region for a left-tailed test with $\alpha = 0.01$.



1% Level of Significance

Example 8: Finding Critical Values for Two-tailed Test

Find a critical value and rejection region for a two tailed test with $\alpha = 0.05$

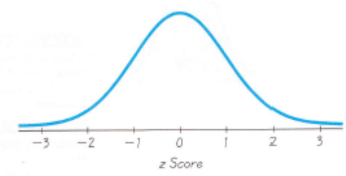


5% Level of Significance

7-2 continued... Finding Rejection Regions

Alpha	Tail	z
0.10	Left	−1.28
	Right	1.28
	Two	±1.645
0.05	Left	−1.645
	Right	1.645
	Two	±1.96
0.01	Left	−2.33
	Right	2.33
	Two	±2.575

This lists COMMON values, not every one you may need - know how to calculate these!

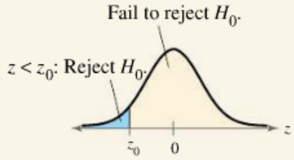


Using the rejection region, we can decide what to do with our null hypothesis

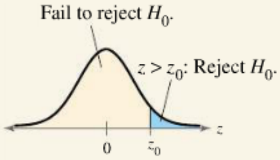
DECISION RULE BASED ON REJECTION REGION

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic z. If the standardized test statistic

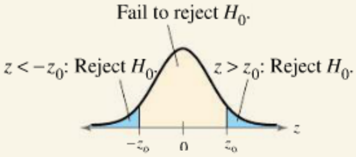
1. is in the rejection region, then reject H_0.
2. is *not* in the rejection region, then fail to reject H_0.



Left-Tailed Test



Right-Tailed Test



Two-Tailed Test

Remember, failing to reject the null hypothesis does not mean that you have accepted the null hypothesis as true. It simply means that there is not enough evidence to reject the null hypothesis.

GUIDELINES

Using Rejection Regions for a z-Test for a Mean μ (σ Known)

IN WORDS	IN SYMBOLS
1. Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the critical value(s).	Use Table 4 in Appendix B.
5. Determine the rejection region(s).	
6. Find the standardized test statistic and sketch the sampling distribution.	$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

7-2 continued... Large-sample Hypothesis Testing

Example 9: Testing population mean with a large sample

Employees in a large accounting firm claim that the mean salary of the firm's accountants is less than that of its competitor's, which is $45,000. A random sample of 30 of the firm's accountants has a mean salary of $43,500 with a standard deviation of $5200. At $\alpha = 0.05$, test the employees' claim.

Solution:

1. Write the hypotheses.

2. Use invnorm or table to find the critical values.

3. Sketch the distribution and label the critical values and rejection regions.

4. Find the z-score of the test statistic and label it on the distribution.

5. Decide to reject or fail to reject the null. Interpret the result.

Example 10: Testing population mean with a large sample

The U.S. Dept. of Agriculture reports that the mean cost of raising a child from birth to age 2 in a rural area is $10,460. You believe this value is incorrect, so you select a random sample of 900 children (age 2) and find that the mean cost is $10,345 with a standard deviation of $1540. At $\alpha = 0.05$, is there enough evidence to conclude that the mean cost is different from $10,460?

Solution:

1. Write the hypotheses.

2. Use invnorm or table to find the critical values.

3. Sketch the distribution and label the critical values and rejection regions.

4. Find the z-score of the test statistic and label it on the distribution.

5. Decide to reject or fail to reject the null. Interpret the result.

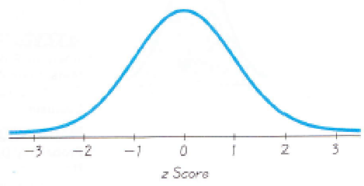
7.2 More examples from the book

EXAMPLE 9

See TI-84 Plus steps on page 415.

Hypothesis Testing Using a Rejection Region

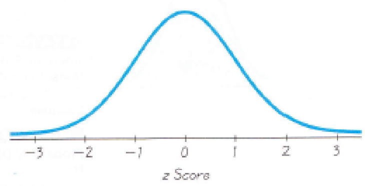
Employees at a construction and mining company claim that the mean salary of the company's mechanical engineers is less than that of one of its competitors, which is \$68,000. A random sample of 20 of the company's mechanical engineers has a mean salary of \$66,900. Assume the population standard deviation is \$5500 and the population is normally distributed. At $\alpha = 0.05$, test the employees' claim.



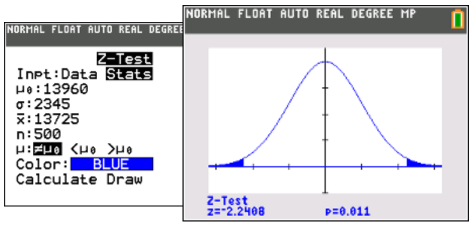
Try It Yourself 9

The CEO of the company in Example 9 claims that the mean work day of the company's mechanical engineers is less than 8.5 hours. A random sample of 25 of the company's mechanical engineers has a mean work day of 8.2 hours. Assume the population standard deviation is 0.5 hour and the population is normally distributed. At $\alpha = 0.01$, test the CEO's claim.

a. Identify the claim and state H_0 and H_a.
b. Identify the level of significance α.
c. Find the critical value z_0 and identify the rejection region.
d. Find the standardized test statistic z. Sketch a graph.
e. Decide whether to reject the null hypothesis.
f. Interpret the decision in the context of the original claim.



EXAMPLE 10 Calc

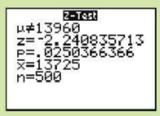


Hypothesis Testing Using Rejection Regions

A researcher claims that the mean annual cost of raising a child (age 2 and under) by husband-wife families in the U.S. is \$13,960. In a random sample of husband-wife families in the U.S., the mean annual cost of raising a child (age 2 and under) is \$13,725. The sample consists of 500 children. Assume the population standard deviation is \$2345. At $\alpha = 0.10$, is there enough evidence to reject the claim?

Study Tip

You can also use technology to perform a hypothesis test using a z-test. For instance, using a TI-84 Plus and the descriptive statistics in Example 10, you can obtain the standardized test statistic $z \approx -2.24$, as shown below. This result matches what you found in Example 10.



pg. 373 3,5,7,10,12, 14,16, 18,19, 21,24
25, 27, 30, 31, 39

Chapter 7: Hypothesis Testing with One Sample

7-3: Hypothesis Tests for the Mean when the population Standard deviation is not known *** USE THIS WHEN Sigma is known *** (σ Unknown)

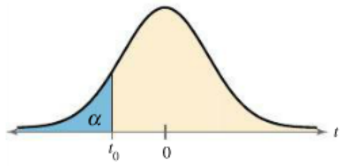
As long as it is still roughly normal, we can still do a hypothesis test - welcome back, t-distribution!

GUIDELINES

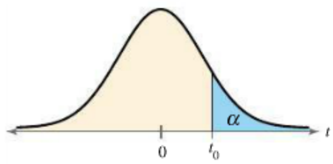
Finding Critical Values in a *t*-Distribution

1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. $= n - 1$.
3. Find the critical value(s) using Table 5 in Appendix B in the row with $n - 1$ degrees of freedom. When the hypothesis test is
   a. *left-tailed,* use the "One Tail, α" column with a negative sign.
   b. *right-tailed,* use the "One Tail, α" column with a positive sign.
   c. *two-tailed,* use the "Two Tails, α" column with a negative and a positive sign.

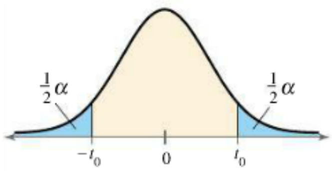
See the figures at the left.



Left-Tailed Test



Right-Tailed Test



Two-Tailed Test

EXAMPLE 1

Finding a Critical Value for a Left-Tailed Test

Find the critical value t_0 for a left-tailed test with $\alpha = 0.05$ and $n = 21$.

Example 2

Find the critical value for a right-tailed test with a significance of .01 and n =17

Table 5— *t*-Distribution

	Level of confidence, *c*	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
25		1.316	1.708	2.060	2.485	2.787

Example 3

Find the critical values $-t_0$ and t_0 for a two-tailed test with $\alpha = 0.10$ and $n = 26$.

7-3 continued... Hypothesis Testing (σ Unknown)

Once we can find the critical value for t, we are basically doing the same type of test as before, when Sigma was known

t-TEST FOR A MEAN μ

The ***t*-test for a mean μ** is a statistical test for a population mean. The **test statistic** is the sample mean $\bar{x}$. The **standardized test statistic** is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$ Standardized test statistic for μ (σ unknown)

when these conditions are met.

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or $n \geq 30$.

The degrees of freedom are

$$\text{d.f.} = n - 1.$$

GUIDELINES

Using the *t*-Test for a Mean μ (σ Unknown)

IN WORDS	IN SYMBOLS
1. Verify that σ is not known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom.	d.f. $= n - 1$
5. Determine the critical value(s).	Use Table 5 in Appendix B.
6. Determine the rejection region(s).	
7. Find the standardized test statistic and sketch the sampling distribution.	$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
8. Make a decision to reject or fail to	If t is in the rejection region,

EXAMPLE 4

See Minitab steps on page 414.

Hypothesis Testing Using a Rejection Region

A used car dealer says that the mean price of a two-year-old sedan (in good condition) is at least \$20,500. You suspect this claim is incorrect and find that a random sample of 14 similar vehicles has a mean price of \$19,850 and a standard deviation of \$1084. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed.

7-3 continued... Hypothesis Testing (σ Unknown)

Example 5: Testing the mean with a small sample

An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 39 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.35, respectively. Is there enough evidence to reject the company's claim at $\alpha = .05$? Assume the population is normally distributed.

Solution:

1) Write the claim, and the null and alternative hypotheses.

2) Decide if it is left, right, or two tailed. Find the critical value for t using the chart.

3) Sketch the distribution and determine the rejection region.

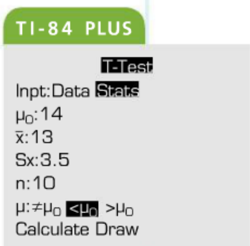
4) Calculate your standardized test statistic.

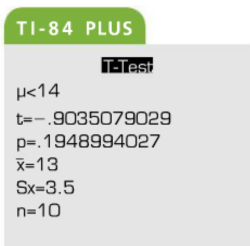
5) Make a decision based on which region it is in and interpret the results.

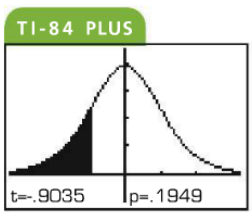
EXAMPLE 6

Using *P*-Values with a *t*-Test

A department of motor vehicles office claims that the mean wait time is less than 14 minutes. A random sample of 10 people has a mean wait time of 13 minutes with a standard deviation of 3.5 minutes. At $\alpha = 0.10$, test the office's claim. Assume the population is normally distributed.







pg. 383: 3,5,7,11,12,14,15,21, 22,23,27

Chapter 7: 7-4: Hypothesis Tests for Proportions

This is going to look very similar to the rest of our hypothesis testing information! Difference is the key words: proportion, percentage, %.

- Used when proportion needs to be tested - for example, when a manufacturer needs to test the proportion of One Direction CDs that are defective.

z-TEST FOR A PROPORTION p

The **z-test for a proportion p** is a statistical test for a population proportion. The z-test can be used when a binomial distribution is given such that $np \geq 5$ and $nq \geq 5$. The **test statistic** is the sample proportion $\hat{p}$ and the **standardized test statistic** is

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}.$$

Standardized test statistic for p

Using a z-Test for a Proportion p

IN WORDS	IN SYMBOLS
1. Verify that the sampling distribution of $\hat{p}$ can be approximated by a normal distribution.	$np \geq 5$, $nq \geq 5$ Check this FIRST! If those conditions are both met, THEN:
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the critical value(s).	Use Table 4 in Appendix B.
5. Determine the rejection region(s).	
6. Find the standardized test statistic and sketch the sampling distribution.	$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$.
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Example 1: Hypothesis Test for a Proportion

Hypothesis Test for a Proportion

A researcher claims that less than 40% of U.S. cell phone owners use their phone for most of their online browsing. In a random sample of 100 adults, 31% say they use their phone for most of their online browsing. At $\alpha = 0.01$, is there enough evidence to support the researcher's claim?

1) Check np and nq

2) Write the claim and the null/alternative hypotheses

3) Determine if it is left, right, or two-tailed and find the critical value for the level of significance.

$$\alpha = .01$$

4) Sketch the distribution and label the rejection regions.

5) Calculate the standardized test statistic

6) Make a decision and interpret the result.

7-4 continued... Proportion Hypothesis Testing

Example 2: Hypothesis Test for a Proportion

Hypothesis Test for a Proportion

A researcher claims that 86% of college graduates say their college degree has been a good investment. In a random sample of 1000 graduates, 845 say their college degree has been a good investment. At $\alpha = 0.10$, is there enough evidence to reject the researcher's claim?

Solution:

Example 2: Hypothesis Test for a Proportion

The Pew Research Center claims that more than 55% of US adults regularly watch their local television news. You decide to test this claim and ask a random sample of 425 adults in the United States whether they regularly watch their local television news. Of the 425 adults, 255 respond yes. At $\alpha = .05$, is there enough evidence to support the claim?

Solution:

pg. 391: 3, 8, 9-11, 15 OR 16
17

7-5 Hypothesis Testing for variance and standard Deviation

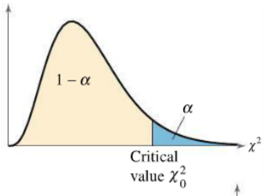
Critical values for a Chi-Square Test

GUIDELINES

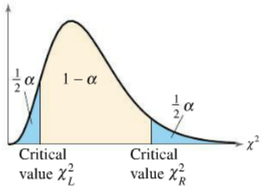
Finding Critical Values for a Chi-Square Test

1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. $= n - 1$.
3. The critical values for the chi-square distribution are found in Table 6 in Appendix B. To find the critical value(s) for a
   a. *right-tailed test,* use the value that corresponds to d.f. and α.
   b. *left-tailed test,* use the value that corresponds to d.f. and $1 - \alpha$.
   c. *two-tailed test,* use the values that correspond to d.f. and $\frac{1}{2}\alpha$, and d.f. and $1 - \frac{1}{2}\alpha$.

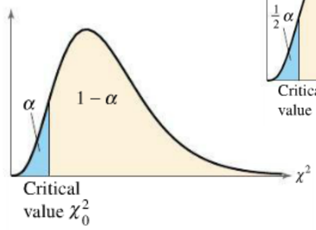
See the figures at the left.



Right-Tailed Test



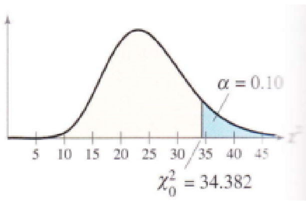
Two-Tailed Test



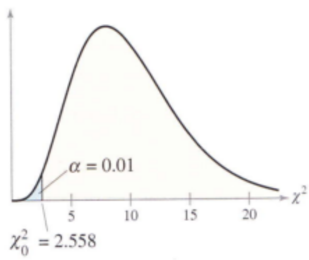
Left-Tailed Test

Example 1 Finding critical values for

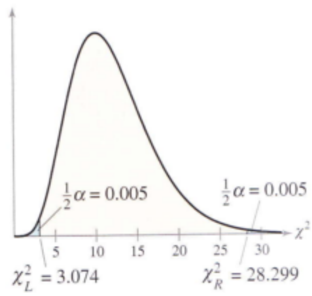
Find the Critical χ^2 value for a right-tailed test when $n = 26 \, and \, \alpha = .10$



Example 2 Finding the critical χ^2 value for a left-tailed test when $n = 11 \; and \; \alpha = .01$



Example 3 Find the critical χ^2 value for a two-tailed test when $n = 13 \; and \; \alpha = .01$



VERY IMPORTANT because Chi-square distributions are not symmetric like normal or t-distributions, in a two-tailed test the two critical values are not opposites you must calculate each of them

7-5 Continued The Chi-Square TEST

To test the variance σ^2 or a standard deviation σ of a population that is a normally distributed, you can use the

The results can be misleading if the population is not normal

CHI-SQUARE TEST FOR A VARIANCE σ^2 OR STANDARD DEVIATION σ

The **chi-square test for a variance σ^2 or standard deviation σ** is a statistical test for a population variance or standard deviation. The chi-square test can only be used when the population is normal. The **test statistic** is s^2 and the **standardized test statistic**

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$ Standardized test statistic for σ^2 or σ

follows a chi-square distribution with degrees of freedom

d.f. $= n - 1$.

Using the Chi-Square Test for a Variance σ^2 or a Standard Deviation σ

IN WORDS	IN SYMBOLS
1. Verify that the sample is random and the population is normally distributed.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom.	d.f. $= n - 1$
5. Determine the critical value(s).	Use Table 6 in Appendix B.
6. Determine the rejection region(s).	
7. Find the standardized test statistic and sketch the sampling distribution.	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$
8. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection regio[n] then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

Example 4
Using a hypothesis test for the population mean
A dairy processing company claims that the variance of the amount of fat in whole milk processed by the company is no more than 0.25. You suspect this is wrong and find that a normal sample of 41 milk containers has a variance of .27. At a $\sigma = .05$, is there enough evidence to reject the company's claim? Assume the population is normally distributed

Example 5 Using a hypothesis test for the SD
A restaurant claims that the standard deviation in the length of serving times is less than 2.9 minutes. A random sample of 23 serving times has a standard deviation of 2.1 minutes. At $\sigma = .10$ is there enough evidence to support the restaurants claim? Assume the population is normal

7-5 Continued

Using a Hypothesis Test for the Standard Deviation

A company claims that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes. A random sample of 25 incoming telephone calls has a standard deviation of 1.1 minutes. At $\alpha = 0.10$, is there enough evidence to support the company's claim? Assume the population is normally distributed.

Example 6 Using a Hypothesis test for the population variance

A sporting goods manufacturer claims that the variance of the strength in a certain fishing line is 15.9. A random sample of 15 fishing line spools has a variance of 21.8. At $\sigma = .05$ is there enough evidence to reject the manufacturer's claim? Assume normal

pg. 400 5,7,9,12,13,15,18,19
21,23

Chapter 7 Summary

With hypothesis testing, perhaps more than any other area of statistics, it can be difficult to see the forest for all the trees. To help you see the forest—the overall picture—a summary of what you studied in this chapter is provided.

You are given a claim about a population parameter μ, p, σ^2, or σ.

Rewrite the claim and its complement using $\underbrace{\le, \ge, =}_{H_0}$ and $\underbrace{>, <, \ne}_{H_a}$.

Identify the claim. Is it H_0 or H_a?

Specify α, the maximum acceptable probability of rejecting a valid H_0

Specify your sample size n.

▲ Normally distributed population ● Any population

Mean: H_0 describes a hypothesized population mean μ.

- ▲ Use a ***z*-test** when σ is known and the population is normal.
- ● Use a ***z*-test** for any population when σ is known and $n \ge 30$.
- ▲ Use a ***t*-test** when σ is not known and the population is normal.
- ● Use a ***t*-test** for any population when σ is not known and $n \ge 30$.

Proportion: H_0 describes a hypothesized population proportion p.

- ● Use a ***z*-test** for any population when $np \ge 5$ and $nq \ge 5$.

Variance or Standard Deviation: H_0 describes a hypothesized population variance σ^2 or standard deviation σ.

- ▲ Use a **chi-square test** when the population is normal.

Use H_a to decide whether the test is left-tailed, right-tailed, or two-tailed.

Take a random sample of size n from the population.

Compute the test statistic $\bar{x}$, $\hat{p}$, or s^2.

Find the standardized test statistic z, t, or χ^2.

Option 1. Decision based on rejection region

Use α to find the critical value(s) z_0, t_0, or χ^2_0 and rejection region(s).

Decision Rule:

Reject H_0 when the standardized test statistic is in the rejection region.
Fail to reject H_0 when the standardized test statistic is not in the rejection region.

Option 2. Decision based on P-value

Use the standardized test statistic or technology to find the P-value.

Decision Rule:

Reject H_0 when $P \le \alpha$.
Fail to reject H_0 when $P > \alpha$.

Chapter 7 Section by section

z-Test for a Hypothesized Mean μ (σ Known) *(Section 7.2)*

Test statistic: $\bar{x}$

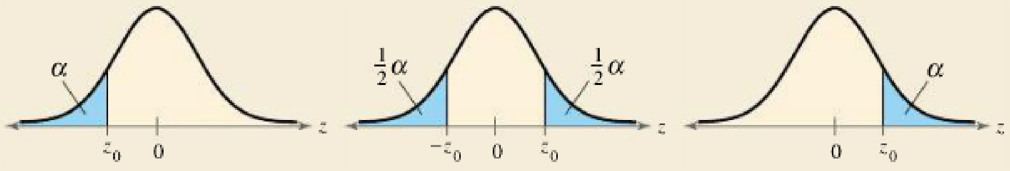
Critical value: z_0 (Use Table 4.)

Sampling distribution of sample means is a normal distribution.

Standardized test statistic: z

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Sample mean ($\bar{x}$); Hypothesized mean (μ); Population standard deviation (σ); Sample size (n)



Left-Tailed | Two-Tailed | Right-Tailed

t-Test for a Hypothesized Mean μ (σ Unknown) *(Section 7.3)*

Test statistic: $\bar{x}$

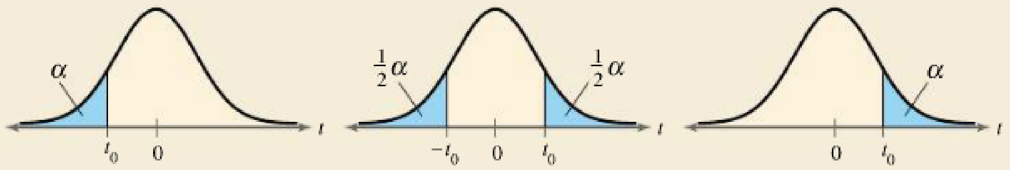
Critical value: t_0 (Use Table 5.)

Sampling distribution of sample means is approximated by a t-distribution with d.f. $= n - 1$.

Standardized test statistic: t

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Sample mean ($\bar{x}$); Hypothesized mean (μ); Sample standard deviation (s); Sample size (n)



Left-Tailed | Two-Tailed | Right-Tailed

z-Test for a Hypothesized Proportion p *(Section 7.4)*

Test statistic: $\hat{p}$

Critical value: z_0 (Use Table 4.)

Sampling distribution of sample proportions is a normal distribution.

Standardized test statistic: z

$$z = \frac{\hat{p} - p}{\sqrt{pq / n}}$$

Sample proportion ($\hat{p}$); Hypothesized proportion (p); $q = 1 - p$; Sample size (n)

Chi-Square Test for a Hypothesized Variance σ^2 or Standard Deviation σ *(Section 7.5)*

Test statistic: s^2

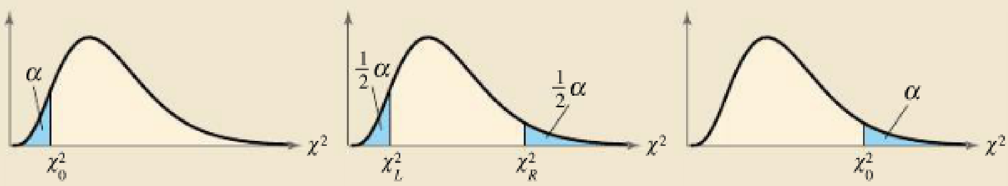
Critical value: χ^2_0 (Use Table 6.)

Sampling distribution is approximated by a chi-square distribution with d.f. $= n - 1$.

Standardized test statistic: χ^2

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Sample size (n); Sample variance (s^2); Hypothesized variance (σ^2)



Left-Tailed | Two-Tailed | Right-Tailed

Calculator Help for Chapter 7

See Example 9, page 371.

TI-84 PLUS

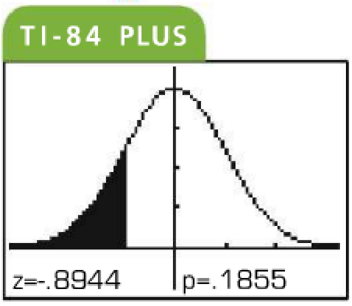
EDIT CALC TESTS
1: Z–Test...
2: T–Test...
3: 2–SampZTest...
4: 2–SampTTest...
5: 1–PropZTest...
6: 2–PropZTest...
7↓ ZInterval...

TI-84 PLUS

Z-Test
Inpt:Data Stats
μ_0:68000
σ:5500
$\bar{x}$:66900
n:20
μ:$\neq\mu_0$ $<\mu_0$ $>\mu_0$
Calculate Draw

TI-84 PLUS

Z-Test
$\mu<68000$
z=−.894427191
p=.1855466488
$\bar{x}$=66900
n=20



See Example 5, page 381.

TI-84 PLUS

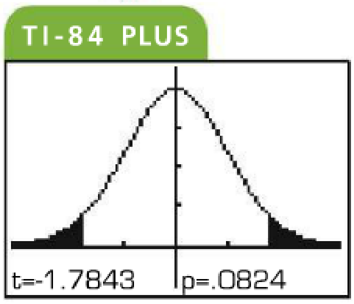
EDIT CALC TESTS
1: Z–Test...
2: T–Test...
3: 2–SampZTest...
4: 2–SampTTest...
5: 1–PropZTest...
6: 2–PropZTest...
7↓ ZInterval...

TI-84 PLUS

T-Test
Inpt:Data Stats
μ_0:6.8
$\bar{x}$:6.7
Sx:.35
n:39
μ:$\neq\mu_0$ $<\mu_0$ $>\mu_0$
Calculate Draw

TI-84 PLUS

T-Test
$\mu\neq6.8$
t=−1.784285142
p=.0823638462
$\bar{x}$=6.7
Sx=.35
n=39



See Example 1, page 389.

TI-84 PLUS

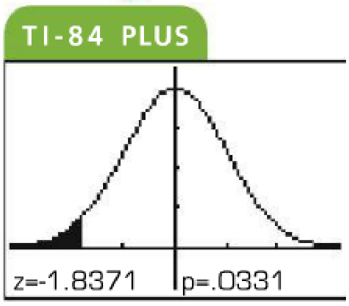
EDIT CALC TESTS
1: Z–Test...
2: T–Test...
3: 2–SampZTest...
4: 2–SampTTest...
5: 1–PropZTest...
6: 2–PropZTest...
7↓ ZInterval...

TI-84 PLUS

1-PropZTest
p_0:.4
x:31
n:100
prop$\neq p_0$ $<p_0$ $>p_0$
Calculate Draw

TI-84 PLUS

1-PropZTest
prop<.4
z=−1.837117307
p=.0330962301
$\hat{p}$=.31
n=100



7-1 Book.pdf

7-2 Book.pdf

7-3 Book.pdf

7-4 Book.pdf

7-5 Book.pdf

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chi-square table .pdf

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