Section P.1
Algebraic Expressions, Mathematical Models, and Real Numbers

It costs how much?

You are looking ahead to the next school year and wondering how much money you will need.
Is there any way that you can use trends for college costs over the past few years to predict how much college will cost next year?
In the Exercise Set for this section, you will use a model that will allow you to project average costs at private U.S. colleges in the near future.

Objective #1: Evaluate algebraic expressions.

✔ *Solved Problem #1*

1. Evaluate $8+6(x-3)^2$ for $x = 13$.

$$\begin{aligned}8+6(x-3)^2 &= 8+6(13-3)^2\\ &= 8+6(10)^2\\ &= 8+6(100)\\ &= 8+600\\ &= 608\end{aligned}$$

✎ *Pencil Problem #1* ✎

1. Evaluate $4+5(x-7)^3$ for $x = 9$.

Objective #2: Use mathematical models.

✔ *Solved Problem #2*

2. The formula $T = 4x^2 + 341x + 3194$ models the average cost of tuition and fees, T, for public U.S. colleges for the school year ending x years after 2000. Use this formula to project the average cost of tuition and fees at public U.S. colleges for the school year ending in 2015.

Because 2015 is 15 years after 2000, we substitute 15 for x in the formula.

$$T = 4x^2 + 341x + 3194$$
$$T = 4(15)^2 + 341(15) + 3194$$
$$T = 4(225)^2 + 341(15) + 3194$$
$$T = 900 + 5115 + 3194$$
$$T = 9209$$

The formula indicates that for the school year ending in 2015, the average cost of tuition and fees at public U.S. colleges will be \$9209.

✎ *Pencil Problem #2* ✎

2. The formula $T = 26x^2 + 819x + 15{,}527$ models the average cost of tuition and fees, T, for private U.S. colleges for the school year ending x years after 2000. Use this formula to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2013.

Objective #3: Find the intersection of two sets.

✔ *Solved Problem #3*

3. Find the intersection: $\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\}$.

The elements common to $\{3, 4, 5, 6, 7\}$ and $\{3, 7, 8, 9\}$ are 3 and 7.

$\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\} = \{3, 7\}$

✎ *Pencil Problem #3* ✐

3. Find the intersection: $\{1, 2, 3, 4\} \cap \{2, 4, 5\}$.

Objective #4: Find the union of two sets.

✔ *Solved Problem #4*

4. Find the union: $\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\}$.

List the elements from the first set: 3, 4, 5, 6, and 7. Now list any elements from the second set not in the first: 8 and 9.

$\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\} = \{3, 4, 5, 6, 7, 8, 9\}$

✎ *Pencil Problem #4* ✐

4. Find the union: $\{1, 2, 3, 4\} \cup \{2, 4, 5\}$.

Objective #5: Recognize subsets of the real numbers..

✔ *Solved Problem #5*

5. Consider the following set of numbers:
$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}$.

5a. List the natural numbers.

The natural numbers are used for counting. The only natural number is $\sqrt{9}$ because $\sqrt{9} = 3$.

5b. List the rational numbers.

All numbers that can be expressed as quotients of integers are rational numbers: $-9\left(-9 = \frac{-9}{1}\right)$, $0\left(0 = \frac{0}{1}\right)$, and $\sqrt{9}\ \left(\sqrt{9} = \frac{3}{1}\right)$. All numbers that are terminating or repeating decimals are rational numbers: -1.3 and $0.\overline{3}$.

✎ *Pencil Problem #5* ✐

5. Consider the following set of numbers:
$\left\{-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\right\}$.

5a. List the natural numbers.

5b. List the rational numbers.

Objective #6: Use inequality symbols.

✔ *Solved Problem #6*

6. Indicate whether each statement is true or false.

6a. $-8 > -3$

This statement is false. Because -8 lies to the left of -3 on a number line, -8 is less than -3. So, $-8 < -3$.

6b. $9 \le 9$

This statement is true because $9 = 9$.

✎ *Pencil Problem #6* ✎

6. Indicate whether each statement is true or false.

6a. $-7 < -2$

6b. $-5 \ge 2$

Objective #7: Evaluate absolute value.

✔ *Solved Problem #7*

7. Rewrite each expression without absolute value bars.

7a. $|1-\sqrt{2}|$

Because $\sqrt{2} \approx 1.4$, the number $1-\sqrt{2}$ is negative. Thus, $|1-\sqrt{2}| = -(1-\sqrt{2}) = \sqrt{2}-1$.

7b. $|\pi - 3|$

Because $\pi \approx 3.14$, the number $\pi - 3$ is positive. Thus, $|\pi - 3| = \pi - 3$.

7c. $\dfrac{|x|}{x}$ if $x > 0$

If $x > 0$, then $|x| = x$. Thus, $\dfrac{|x|}{x} = \dfrac{x}{x} = 1$.

✎ *Pencil Problem #7* ✎

7. Rewrite each expression without absolute value bars.

7a. $|12 - \pi|$

7b. $|\sqrt{2} - 5|$

7c. $\dfrac{-3}{|-3|}$

Objective #8: Use absolute value to express distance.

✔ *Solved Problem #8*

8. Find the distance between -4 and 5 on the real number line.

$|-4-5| = |-9| = 9$

✎ *Pencil Problem #8* ✎

8. Find the distance between -19 and -4 on the real number line.

Objective #9: Identify properties of the real numbers.

✔ *Solved Problem #9*

9. State the name of the property illustrated.

9a. $2+\sqrt{5}=\sqrt{5}+2$

The order of the numbers in the addition has changed. This illustrates the commutative property of addition.

9b. $1\cdot(x+3)=x+3$

One has been deleted from a product. This illustrates the identity property of multiplication.

✎ *Pencil Problem #9* ✐

9. State the name of the property illustrated.

9a. $6+(2+7)=(6+2)+7$

9b. $2(-8+6)=-16+12$

Objective #10: Simplify algebraic expressions.

✔ *Solved Problem #10*

10. Simplify: $6+4[7-(x-2)]$.

$$\begin{aligned} 6+4[7-(x-2)] &= 6+4[7-x+2] \\ &= 6+4[9-x] \\ &= 6+36-4x \\ &= (6+36)-4x \\ &= 42-4x \end{aligned}$$

✎ *Pencil Problem #10* ✐

10. Simplify: $7-4[3-(4y-5)]$.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. 44 *(P.1 #9)* **2.** \$30,568 *(P.1 #131c)* **3.** $\{2, 4\}$ *(P.1 #21)* **4.** $\{1, 2, 3, 4, 5\}$ *(P.1 #29)*
5. a. $\sqrt{64}$ **b.** $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$ *(P.1 #37)* **6. a.** true **b.** false **7. a.** $12-\pi$ *(P.1 #53)*
b. $5-\sqrt{2}$ *(P.1 #55)* **c.** -1 *(P.1 #57)* **8.** 15 *(P.1 #71)* **9. a.** associative property of addition *(P.1 #77)*
b. distributive property *(P.1 #81)* **10.** $16y-25$ *(P.1 #93)*

Section P.2
Exponents and Scientific Notation

WOW, THAT'S BIG!

Did you know that in the summer of 2012 the national debt passed \$16,000,000,000,000 or \$16 trillion? Yes, that's 12 zeros you count. In this section, you will express the national debt in a form called *scientific notation* and use this form to calculate your share of the debt.

Objective #1: Use the product rule.

✔ *Solved Problem #1*

1. Multiply using the product rule.

1a. $3^3 \cdot 3^2$

$3^3 \cdot 3^2 = 3^{3+2} = 3^5$ or 243

1b. $(4x^3y^4)(10x^2y^6)$

$(4x^3y^4)(10x^2y^6) = 4 \cdot 10 \cdot x^{3+2} \cdot y^{4+6} = 40x^5y^{10}$

✎ *Pencil Problem #1* ✎

1. Multiply using the product rule.

1a. $x^3 \cdot x^7$

1b. $(-9x^3y)(-2x^6y^4)$

Objective #2: Use the quotient rule.

✔ *Solved Problem #2*

2. Divide using the quotient rule.

2a. $\dfrac{(-3)^6}{(-3)^3}$

$\dfrac{(-3)^6}{(-3)^3} = (-3)^{6-3} = (-3)^3$ or -27

2b. $\dfrac{27x^{14}y^8}{3x^3y^5}$

$\dfrac{27x^{14}y^8}{3x^3y^5} = \dfrac{27}{3} \cdot x^{14-3} \cdot y^{8-5} = 9x^{11}y^3$

✎ *Pencil Problem #2* ✎

2. Divide using the quotient rule.

2a. $\dfrac{2^8}{2^4}$

2b. $\dfrac{25a^{13}b^4}{-5a^2b^3}$

Objective #3: Use the zero-exponent rule.

✔ *Solved Problem #3*

3. Evaluate -8^0.

Because there are no parentheses only 8 is raised to the 0 power.
$-8^0 = -(8^0) = -1$

✎ *Pencil Problem #3* ✐

3. Evaluate $(-3)^0$.

Objective #4: Use the negative-exponent rule.

✔ *Solved Problem #4*

4. Write with a positive exponent. Simplify, if possible.

4a. 5^{-2}

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

4b. $3x^{-6}y^4$

$3x^{-6}y^4 = 3 \cdot \frac{1}{x^6} \cdot y^4 = \frac{3y^4}{x^6}$

✎ *Pencil Problem #4* ✐

4. Write with a positive exponent. Simplify, if possible.

4a. 4^{-3}

4b. $(4x^3)^{-2}$

Objective #5: Use the power rule.

✔ *Solved Problem #5*

5. Simplify using the power rule.

5a. $(3^3)^2$

$(3^3)^2 = 3^{3 \cdot 2} = 3^6$ or 729

✎ *Pencil Problem #5* ✐

5. Simplify using the power rule.

5a. $(2^2)^3$

5b. $(y^7)^{-2}$

$$(y^7)^{-2} = y^{7(-2)} = y^{-14} = \frac{1}{y^{14}}$$

5b. $(x^{-5})^3$

Objective #6: Find the power of a product.

Solved Problem #6

6. Simplify: $(-4x)^3$.

$$(-4x)^3 = (-4)^3(x)^3 = -64x^3$$

Pencil Problem #6

6. Simplify: $(8x^3)^2$.

Objective #7: Find the power of a quotient.

Solved Problem #7

7. Simplify: $\left(-\frac{2}{y}\right)^5$.

$$\left(-\frac{2}{y}\right)^5 = \frac{(-2)^5}{y^5} = \frac{-32}{y^5} = -\frac{32}{y^5}$$

Pencil Problem #7

7. Simplify: $\left(-\frac{4}{x}\right)^3$.

Objective #8: Simplify exponential expressions.

Solved Problem #8

8. Simplify.

8a. $(2x^3y^6)^4$

$$\begin{aligned}(2x^3y^6)^4 &= (2)^4(x^3)^4(y^6)^4\\ &= 2^4x^{3\cdot 4}y^{6\cdot 4}\\ &= 16x^{12}y^{24}\end{aligned}$$

Pencil Problem #8

8. Simplify.

8a. $(-3x^2y^5)^2$

8b. $(-6x^2y^5)(3xy^3)$

$$\begin{aligned}(-6x^2y^5)(3xy^3) &= (-6)(3)x^2xy^5y^3\\ &= -18x^{2+1}y^{5+3}\\ &= -18x^3y^8\end{aligned}$$

8b. $(3x^4)(2x^7)$

8c. $\dfrac{100x^{12}y^2}{20x^{16}y^{-4}}$

$$\begin{aligned}\frac{100x^{12}y^2}{20x^{16}y^{-4}} &= \left(\frac{100}{20}\right)\left(\frac{x^{12}}{x^{16}}\right)\left(\frac{y^2}{y^{-4}}\right)\\ &= 5x^{12-16}y^{2-(-4)}\\ &= 5x^{-4}y^6\\ &= \frac{5y^6}{x^4}\end{aligned}$$

8c. $\dfrac{24x^3y^5}{32x^7y^{-9}}$

8d. $\left(\dfrac{5x}{y^4}\right)^{-2}$

$$\begin{aligned}\left(\frac{5x}{y^4}\right)^{-2} &= \frac{(5x)^{-2}}{(y^4)^{-2}}\\ &= \frac{5^{-2}x^{-2}}{y^{-6}}\\ &= \frac{y^6}{5^2x^2}\\ &= \frac{y^6}{25x^2}\end{aligned}$$

8d. $\left(\dfrac{5x^3}{y}\right)^{-2}$

Objective #9: Use scientific notation.

Solved Problem #9

9. In 9a and 9b, write each number in decimal notation.

9a. -2.6×10^{9}

Move the decimal point 9 places to the right.
$-2.6\times10^{9}=-2,600,000,000$

9b. 3.017×10^{-6}

Move the decimal point 6 places to the left.
$3.017\times10^{-6}=0.000003017$

9c. In 9c and 9d, write each number in scientific notation.

5,210,000,000

The decimal point needs to be moved 9 places to the left.
$5,210,000,000=5.21\times10^{9}$

9d. -0.000000006893

The decimal point needs to be moved 8 places to the right.
$-0.00000006893=-6.893\times10^{-8}$

Pencil Problem #9

9. In 9a and 9b, write each number in decimal notation.

9a. -7.16×10^{6}

9b. 7.9×10^{-1}

9c. In 9c and 9d, write each number in scientific notation.

32,000

9b. -0.00000000504

9e. In 9e and 9f, perform the indicated computations. Write the answers in scientific notation.

$(7.1\times10^{5})(5\times10^{-7})$

$$\begin{aligned}(7.1\times10^{5})(5\times10^{-7}) &= (7.1\times5)\times10^{5+(-7)}\\ &= 35.5\times10^{-2}\\ &= (3.55\times10^{1})\times10^{-2}\\ &= 3.55\times10^{-1}\end{aligned}$$

9f. $\dfrac{1.2\times10^{6}}{3\times10^{-3}}$

$$\begin{aligned}\frac{1.2\times10^{6}}{3\times10^{-3}} &= \frac{1.2}{3}\times10^{6-(-3)}\\ &= 0.4\times10^{9}\\ &= (4\times10^{-1})\times10^{9}\\ &= 4\times10^{8}\end{aligned}$$

9e. In 9e and 9f, perform the indicated computations. Write the answers in scientific notation.

$(1.6\times10^{15})(4\times10^{-11})$

9f. $\dfrac{2.4\times10^{-2}}{4.8\times10^{-6}}$

<u>**Answers**</u> <u>for Pencil Problems</u> <u>*(Textbook Exercise references in parentheses)*</u>:

1a. x^{10} *(P.2 #27)* **1b.** $18x^{9}y^{5}$ *(P.2 #47)* **2a.** 16 *(P.2 #17)* **2b.** $-5a^{11}b$ *(P.2 #51)*

3. 1 *(P.2 #7)* **4a.** $\dfrac{1}{64}$ *(P.2 #11)* **4b.** $\dfrac{1}{16x^{6}}$ *(P.2 #55)*

5a. 64 *(P.2 #15)* **5b.** $\dfrac{1}{x^{15}}$ *(P.2 #33)*

6. $64x^{6}$ *(P.2 #39)* **7.** $-\dfrac{64}{x^{3}}$ *(P.2 #41)*

8a. $9x^{4}y^{10}$ *(P.2 #43)* **8b.** $6x^{11}$ *(P.2 #45)* **8c.** $\dfrac{3y^{14}}{4x^{4}}$ *(P.2 #57)* **8d.** $\dfrac{y^{2}}{25x^{6}}$ *(P.2 #59)*

9a. –7,160,000 *(P.2 #69)* **9b.** 0.79 *(P.2 #71)* **9c.** 3.2×10^{4} *(P.2 #77)* **9d.** -5.04×10^{-9} *(P.2 #85)*
9e. 6.4×10^{4} *(P.2 #89)* **9f.** 5×10^{3} *(P.2 #101)*

Section P.3
Radicals and Rational Exponents

Radicals in Space?

What does space travel have to do with radicals?

Imagine that in the future we will be able to travel at velocities approaching the speed of light
(approximately 186,000 miles per second). According to Einstein's theory of special relativity, time would pass more quickly on Earth than it would in the moving spaceship.

Objective #1: Evaluate square roots.	
✔ ***Solved Problem #1***	***Pencil Problem #1***
1a. Evaluate $\sqrt{81}$. $\sqrt{81}=9$ Check: $9^2=81$	**1a.** Evaluate $\sqrt{36}$.
1b. Evaluate $-\sqrt{9}$. $-\sqrt{9}=-3$ Check: $(-3)^2=9$	**1b.** Evaluate $-\sqrt{36}$.
1c. Evaluate $\sqrt{\frac{1}{25}}$. $\sqrt{\frac{1}{25}}=\frac{1}{5}$ Check: $\left(\frac{1}{5}\right)^2=\frac{1}{25}$	**1c.** Evaluate $\sqrt{\frac{1}{81}}$.
1d. Evaluate $\sqrt{36+64}$. $\sqrt{36+64}=\sqrt{100}=10$	**1d.** Evaluate $\sqrt{25-16}$.

1e. Evaluate $\sqrt{36}+\sqrt{64}$.

$\sqrt{36}+\sqrt{64}=6+8=14$

1e. Evaluate $\sqrt{25}-\sqrt{16}$.

Objective #2: Simplify expressions of the form $\sqrt{a^2}$.

✔ ***Solved Problem #2***

2. Evaluate $\sqrt{(-6)^2}$.

$\sqrt{(-6)^2}=|-6|=6$

✎ ***Pencil Problem #2*** ✐

2. Evaluate $\sqrt{(-13)^2}$.

Objective #3: Use the product rule to simplify square roots.

✔ ***Solved Problem #3***

3a. Simplify $\sqrt{75}$.

$\sqrt{75}=\sqrt{25\cdot 3}=\sqrt{25}\cdot\sqrt{3}=5\sqrt{3}$

3b. Simplify $\sqrt{5x}\cdot\sqrt{10x}$.

$$\begin{aligned}\sqrt{5x}\cdot\sqrt{10x}&=\sqrt{5x\cdot 10x}\\&=\sqrt{50x^2}\\&=\sqrt{25x^2\cdot 2}\\&=\sqrt{25x^2}\sqrt{2}\\&=5x\sqrt{2}\end{aligned}$$

✎ ***Pencil Problem #3*** ✐

3a. Simplify $\sqrt{50}$.

3b. Simplify $\sqrt{2x}\cdot\sqrt{6x}$.

Objective #4: Use the quotient rule to simplify square roots.

✔ ***Solved Problem #4***

4a. Simplify $\sqrt{\frac{25}{16}}$.

$$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$$

4b. Simplify $\frac{\sqrt{150x^3}}{\sqrt{2x}}$.

$$\begin{aligned} \frac{\sqrt{150x^3}}{\sqrt{2x}} &= \sqrt{\frac{150x^3}{2x}} \\ &= \sqrt{75x^2} \\ &= \sqrt{25x^2 \cdot 3} \\ &= \sqrt{25x^2}\sqrt{3} \\ &= 5x\sqrt{3} \end{aligned}$$

✎ ***Pencil Problem #4*** ✎

4a. Simplify $\sqrt{\frac{49}{16}}$.

4b. Simplify $\frac{\sqrt{48x^3}}{\sqrt{3x}}$.

Objective #5: Add and subtract square roots.

✔ ***Solved Problem #5***

5a. Add: $8\sqrt{13}+9\sqrt{13}$.

$$8\sqrt{13}+9\sqrt{13} = (8+9)\sqrt{13} = 17\sqrt{13}$$

✎ ***Pencil Problem #5*** ✎

5a. Subtract: $6\sqrt{17x}-8\sqrt{17x}$.

5b. Subtract: $6\sqrt{18x} - 4\sqrt{8x}$.

$$\begin{aligned} 6\sqrt{18x} - 4\sqrt{8x} &= 6\sqrt{9 \cdot 2x} - 4\sqrt{4 \cdot 2x} \\ &= 6 \cdot 3\sqrt{2x} - 4 \cdot 2\sqrt{2x} \\ &= 18\sqrt{2x} - 8\sqrt{2x} \\ &= (18-8)\sqrt{2x} \\ &= 10\sqrt{2x} \end{aligned}$$

5b. Add: $3\sqrt{18} + 5\sqrt{50}$.

Objective #6: Rationalize denominators.

✔ *Solved Problem #6*

6a. Rationalize the denominator : $\dfrac{6}{\sqrt{12}}$.

Multiply by $\sqrt{3}$ to obtain the square root of a perfect square, $\sqrt{12} \cdot \sqrt{3} = \sqrt{36}$.

$$\frac{6}{\sqrt{12}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{36}} = \frac{6\sqrt{3}}{6} = \sqrt{3}$$

6b. Rationalize the denominator : $\dfrac{8}{4+\sqrt{5}}$.

Multiply by $4-\sqrt{5}$, the conjugate of $4+\sqrt{5}$.

$$\begin{aligned} \frac{8}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}} &= \frac{8(4-\sqrt{5})}{4^2-(\sqrt{5})^2} \\ &= \frac{8(4-\sqrt{5})}{16-5} \\ &= \frac{8(4-\sqrt{5})}{11} \quad \text{or} \quad \frac{32-8\sqrt{5}}{11} \end{aligned}$$

✎ *Pencil Problem #6* ✐

6a. Rationalize the denominator : $\dfrac{\sqrt{2}}{\sqrt{5}}$.

6b. Rationalize the denominator : $\dfrac{7}{\sqrt{5}-2}$.

Objective #7: Evaluate and perform operations with higher roots.

✔ *Solved Problem #7*	*Pencil Problem #7*
7a. Simplify $\sqrt[5]{8}\cdot\sqrt[5]{8}$. $\sqrt[5]{8}\cdot\sqrt[5]{8}=\sqrt[5]{8\cdot 8}$ $=\sqrt[5]{64}$ $=\sqrt[5]{32\cdot 2}$ $=\sqrt[5]{32}\cdot\sqrt[5]{2}$ $=2\sqrt[5]{2}$	**7a.** Simplify $\sqrt[3]{9}\cdot\sqrt[3]{6}$.
7b. Simplify $\sqrt[3]{\frac{125}{27}}$. $\sqrt[3]{\frac{125}{27}}=\frac{\sqrt[3]{125}}{\sqrt[3]{27}}=\frac{5}{3}$	**7b.** Simplify $\frac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}}$.
7c. Subtract: $3\sqrt[3]{81}-4\sqrt[3]{3}$. $3\sqrt[3]{81}-4\sqrt[3]{3}=3\sqrt[3]{27\cdot 3}-4\sqrt[3]{3}$ $=3\cdot 3\sqrt[3]{3}-4\sqrt[3]{3}$ $=9\sqrt[3]{3}-4\sqrt[3]{3}$ $=(9-4)\sqrt[3]{3}$ $=5\sqrt[3]{3}$	**7c.** Add: $5\sqrt[3]{16}+\sqrt[3]{54}$.

Objective #8: Understand and use rational exponents..

✔ *Solved Problem #8*	✎ *Pencil Problem #8* ✎
8a. Simplify $(-8)^{\frac{1}{3}}$. $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$	**8a.** Simplify $36^{\frac{1}{2}}$.
8b. Simplify $32^{-\frac{2}{5}}$. $32^{-\frac{2}{5}} = \dfrac{1}{32^{\frac{2}{5}}} = \dfrac{1}{(\sqrt[5]{32})^2} = \dfrac{1}{2^2} = \dfrac{1}{4}$	**8b.** Simplify $125^{\frac{2}{3}}$.
8c. Simplify $\dfrac{20x^4}{5x^{\frac{3}{2}}}$. $\dfrac{20x^4}{5x^{\frac{3}{2}}} = \dfrac{20}{5} \cdot x^{4-\frac{3}{2}} = 4x^{\frac{8}{2}-\frac{3}{2}} = 4x^{\frac{5}{2}}$	**8c.** Simplify $(7x^{\frac{1}{3}})(2x^{\frac{1}{4}})$.
8d. Simplify $\sqrt[6]{x^3}$. $\sqrt[6]{x^3} = x^{\frac{3}{6}} = x^{\frac{1}{2}} = \sqrt{x}$	**8d.** Simplify $\sqrt[6]{x^4}$.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. 6 *(P.3 #1)* **1b.** -6 *(P.3 #3)* **1c.** $\frac{1}{9}$ *(P.3 #23)* **1d.** 3 *(P.3 #7)* **1e.** 1 *(P.3 #9)*

2. 13 *(P.3 #11)* **3a.** $5\sqrt{2}$ *(P.3 #13)* **3b.** $2x\sqrt{3}$ *(P.3 #17)*

4a. $\frac{7}{4}$ *(P.3 #23)* **4b.** $4x$ *(P.3 #27)* **5a.** $-2\sqrt{17x}$ *(P.3 #35)* **5b.** $34\sqrt{2}$ *(P.3 #41)*

6a. $\frac{\sqrt{10}}{5}$ *(P.3 #47)* **6b.** $7(\sqrt{5}+2)$ *(P.3 #51)*

7a. $3\sqrt[3]{2}$ *(P.3 #71)* **7b.** $2x$ *(P.3 #73)* **7c.** $13\sqrt[3]{2}$ *(P.3 #77)*

8a. 6 *(P.3 #83)* **8b.** 25 *(P.3 #87)* **8c.** $14x^{\frac{7}{12}}$ *(P.3 #91)* **8d.** $\sqrt[3]{x^2}$ *(P.3 #105)*

Section P.4
Polynomials

What Are the Best Dimensions for a Box?

Many children get excited about gift boxes of all shapes and sizes, with the possible *exception* of clothing-sized boxes. (I must confess I dreaded boxes of that size.)

While completing the application exercises in this section of the textbook, we will use polynomials to model the dimensions of a box. We will then apply the concepts of this section to model the area of the box's base and its volume.

Objective #1: Understand the vocabulary of polynomials.

✔ *Solved Problem #1*

1. True or false: $7x^5 - 3x^3 + 8$ is a polynomial of degree 7 with three terms.

False. The expression $7x^5 - 3x^3 + 8$ is a polynomial with three terms, but its degree is 5, not 7.

✎ *Pencil Problem #1* ✎

1. True or false: $x^2 - 4x^3 + 9x - 12x^4 + 63$ is a polynomial of degree 2 with five terms.

Objective #2: Add and subtract polynomials.

✔ *Solved Problem #2*

2a. Add:
$(-17x^3 + 4x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15)$.

$$\begin{aligned}
&(-17x^3 + 4x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15) \\
&= (-17x^3 + 16x^3) + (4x^2 - 3x^2) + (-11x + 3x) + (-5 - 15) \\
&= -x^3 + x^2 + (-8x) + (-20) \\
&= -x^3 + x^2 - 8x - 20
\end{aligned}$$

✎ *Pencil Problem #2* ✎

2a. Add: $(-6x^3 + 5x^2 - 8x + 9) + (17x^3 + 2x^2 - 4x - 13)$.

2b. Subtract:
$(13x^3 - 9x^2 - 7x + 1) - (-7x^3 + 2x^2 - 5x + 9).$

$$\begin{aligned}
&(13x^3 - 9x^2 - 7x + 1) - (-7x^3 + 2x^2 - 5x + 9)\\
&= (13x^3 - 9x^2 - 7x + 1) + (7x^3 - 2x^2 + 5x - 9)\\
&= (13x^3 + 7x^3) + (-9x^2 - 2x^2) + (-7x + 5x) + (1 - 9)\\
&= 20x^3 + (-11x^2) + (-2x) + (-8)\\
&= 20x^3 - 11x^2 - 2x - 8
\end{aligned}$$

2b. Subtract:
$(17x^3 - 5x^2 + 4x - 3) - (5x^3 - 9x^2 - 8x + 11).$

Objective #3: Multiply polynomials.

✔ *Solved Problem #3*

3. Multiply: $(5x - 2)(3x^2 - 5x + 4).$

$$\begin{aligned}
&(5x - 2)(3x^2 - 5x + 4)\\
&= 5x(3x^2 - 5x + 4) - 2(3x^2 - 5x + 4)\\
&= 5x \cdot 3x^2 + 5x(-5x) + 5x \cdot 4 - 2 \cdot 3x^2 - 2(-5x) - 2 \cdot 4\\
&= 15x^3 - 25x^2 + 20x - 6x^2 + 10x - 8\\
&= 15x^3 - 31x^2 + 30x - 8
\end{aligned}$$

✎ *Pencil Problem #3* ✎

3. Multiply: $(2x - 3)(x^2 - 3x + 5).$

Objective #4: Use FOIL in polynomial multiplication.

✔ *Solved Problem #4*

4. Multiply: $(7x - 5)(4x - 3).$
Use FOIL.

First: $7x \cdot 4x$ Outside: $7x(-3)$
Inside: $-5 \cdot 4x$ Last: $-5(-3)$

$$\begin{aligned}
&(7x - 5)(4x - 3)\\
&= 7x \cdot 4x + 7x(-3) - 5 \cdot 4x - 5(-3)\\
&= 28x^2 - 21x - 20x + 15\\
&= 28x^2 - 41x + 15
\end{aligned}$$

✎ *Pencil Problem #4* ✎

4. Multiply: $(3x + 5)(2x + 1).$

Objective #5: Use special products in polynomial multiplication.

✔ Solved Problem #5

5a. Multiply: $(7x+8)(7x-8)$.

Use $(A+B)(A-B)=A^2-B^2$.

$$\begin{aligned}(7x+8)(7x-8)&=(7x)^2-8^2\\&=49x^2-64\end{aligned}$$

5b. Multiply: $(5x+4)^2$.

Use $(A+B)^2=A^2+2AB+B^2$.

$$\begin{aligned}(5x+4)^2&=(5x)^2+2(5x)(4)+4^2\\&=25x^2+40x+16\end{aligned}$$

5c. Multiply: $(x-9)^2$.

Use $(A-B)^2=A^2-2AB+B^2$.

$$\begin{aligned}(x-9)^2&=x^2-2\cdot x\cdot 9+9^2\\&=x^2-18x+81\end{aligned}$$

✎ Pencil Problem #5 ✎

5a. Multiply: $(5-7x)(5+7x)$.

5b. Multiply: $(2x+3)^2$.

5c. Multiply: $(x-3)^2$.

Objective #6: Perform operations with polynomials in several variables.

✔ ***Solved Problem #6***	***Pencil Problem #6***
6a. Subtract: $(x^3-4x^2y+5xy^2-y^3)-(x^3-6x^2y+y^3)$. $(x^3-4x^2y+5xy^2-y^3)-(x^3-6x^2y+y^3)$ $=(x^3-4x^2y+5xy^2-y^3)+(-x^3+6x^2y-y^3)$ $=(x^3-x^3)+(-4x^2y+6x^2y)+5xy^2+(-y^3-y^3)$ $=2x^2y+5xy^2-2y^3$	**6a.** Add: $(4x^2y+8xy+11)+(-2x^2y+5xy+2)$.
6b. Multiply: $(7x-6y)(3x-y)$. Use FOIL. $(7x-6y)(3x-y)$ $=7x\cdot 3x+7x(-y)-6y\cdot 3x-6y(-y)$ $=21x^2-7xy-18xy+6y^2$ $=21x^2-25xy+6y^2$	**6b.** Multiply: $(7x+5y)^2$.

<u>**Answers**</u> <u>for Pencil Problems</u> <u>*(Textbook Exercise references in parentheses)*</u>:

1. False *(P.4 #7)* **2a.** $11x^3+7x^2-12x-4$ *(P.4 #9)* **2b.** $12x^3+4x^2+12x-14$ *(P.4 #11)*
3. $2x^3-9x^2+19x-15$ *(P.4 #17)* **4.** $6x^2+13x+5$ *(P.4 #23)*
5a. $25-49x^2$ *(P.4 #35)* **5b.** $4x^2+12x+9$ *(P.4 #43)* **5c.** x^2-6x+9 *(P.4 #45)*
6a. $2x^2y+13xy+13$ *(P.4 #61)* **6b.** $49x^2+70xy+25y^2$ *(P.4 #73)*

Section P.5
Factoring Polynomials

What's the sales price?

Many times retailers advertise their discounts in terms of percentages by which the price is reduced, such as 30% off. If a product still doesn't sell, the retailer may offer an additional 30% off the price that has already been reduced by 30%.

In this section's Exercise Set, you will see how the 30% discount followed by another 30% discount can be expressed as a polynomial. By factoring the polynomial and simplifying, you will see that our double discount means that we pay 49% of the original price.

Objective #1: Factor out the greatest common factor.

✔ *Solved Problem #1*

1a. Factor $10x^3 - 4x^2$.

2 is the greatest integer that divides 10 and 4. x^2 is the greatest expression that divides x^3 and x^2. The GCF is $2x^2$.

$$\begin{aligned} 10x^3 - 4x^2 &= 2x^2(5x) - 2x^2(2) \\ &= 2x^2(5x-2) \end{aligned}$$

1b. Factor $2x(x-7)+3(x-7)$.

The GCF is the binomial factor $(x-7)$.

$2x(x-7)+3(x-7)=(x-7)(2x+3)$

✎ *Pencil Problem #1* ✎

1a. Factor $3x^2 + 6x$.

1b. Factor $x(x+5)+3(x+5)$.

Objective #2: Factor by grouping.

✔ *Solved Problem #2*

2. Factor $x^3+5x^2-2x-10$.

The GCF of the first two terms is x^2, and the GCF of the last two terms is −2. After factoring out these GCFs, factor out the common binomial factor.

$$\begin{aligned} x^3+5x^2-2x-10 &= (x^3+5x^2)+(-2x-10) \\ &= x^2(x+5)-2(x+5) \\ &= (x+5)(x^2-2) \end{aligned}$$

✎ *Pencil Problem #2* ✎

2. Factor $x^3-2x^2+5x-10$.

Objective #3: Factor trinomials.

✔ *Solved Problem #3*

3a. Factor $x^2-5x-14$.

The leading coefficient is 1. We look for factors of −14 that sum to −5.

−7(2) = −14 and −7 + 2 = −5
The numbers are −7 and 2.
$x^2-5x-14=(x-7)(x+2)$

✎ *Pencil Problem #3* ✎

3a. Factor $x^2-8x+15$.

3b. Factor $6x^2+19x-7$.

The leading coefficient is 6, not 1. $6x^2$ factors as $6x(x)$ or $3x(2x)$. -7 factors as $-7(1)$ or $7(-1)$.

The possible factorizations are

$(6x-7)(x+1)$	$(6x+1)(x-7)$
$(6x+7)(x-1)$	$(6x-1)(x+7)$
$(3x-7)(2x+1)$	$(3x+1)(2x-7)$
$(3x+7)(2x-1)$	$(3x-1)(2x+7)$

We want the combination, if there is one, that results in a sum of Outside and Inside terms of $19x$. Compute the sums of the Outside and Inside terms in the possible factorizations until you find one that results in $19x$.

For $(3x-1)(2x+7)$:
Outside: $3x(7)=21x$
Inside: $-1(2x)=-2x$
Sum: $21x+(-2x)=19x$
So, $6x^2+19x-7=(3x-1)(2x+7)$.

3b. Factor $9x^2-9x+2$.

Objective #4: Factor the difference of squares.

✔ *Solved Problem #4*

4. Factor: $36x^2-25$.

Note that $36x^2=(6x)^2$ and $25=5^2$ can both be expressed as squares.

Use $A^2-B^2=(A+B)(A-B)$.

$$\begin{aligned}36x^2-25&=(6x)^2-5^2\\&=(6x+5)(6x-5)\end{aligned}$$

✎ *Pencil Problem #4* ✐

4. Factor $9x^2-25y^2$.

Objective #5: Factor perfect square trinomials.

✔ *Solved Problem #5*

5a. Factor $x^2 + 14x + 49$.

Note that the first term is the square of *x*, the last term is the square of 7, and the middle term is twice the product of *x* and 7.

Factor using $A^2 + 2AB + B^2 = (A + B)^2$.

$$x^2 + 14x + 49 = x^2 + 2 \cdot x \cdot 7 + 7^2$$
$$= (x + 7)^2$$

5b. Factor $16x^2 - 56x + 49$.

Note that the first term is the square of 4*x*, the last term is the square of 7, and the middle term is twice the product of 4*x* and 7.
Factor using $A^2 - 2AB + B^2 = (A - B)^2$.

$$16x^2 - 56x + 49 = (4x)^2 - 2 \cdot 4x \cdot 7 + 7^2$$
$$= (4x - 7)^2$$

✎ *Pencil Problem #5* ✐

5a. Factor $x^2 + 2x + 1$.

5b. Factor $9x^2 - 6x + 1$.

Objective #6: Factor the sum or difference of two cubes.

✔ *Solved Problem #6*

6a. Factor $x^3 + 1$.

Note that both terms can be expressed as cubes.
Factor using $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$.

$$x^3 + 1 = x^3 + 1^3$$
$$= (x + 1)(x^2 - x \cdot 1 + 1^2)$$
$$= (x + 1)(x^2 - x + 1)$$

✎ *Pencil Problem #6* ✐

6a. Factor $x^3 + 27$.

6b. Factor $125x^3 - 8$.

Note that the first term is the cube of $5x$ and the second term is the cube of 2.

Factor using $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$.

$$\begin{aligned} 125x^3 - 8 &= (5x)^3 - 2^3 \\ &= (5x-2)[(5x)^2 + 5x \cdot 2 + 2^2] \\ &= (5x-2)(25x^2 + 10x + 4) \end{aligned}$$

6b. Factor $8x^3 - 1$.

Objective #7: Use a general strategy for factoring polynomials..

✔ *Solved Problem #7*

7a. Factor $3x^3 - 30x^2 + 75x$.

First, factor out the GCF, $3x$.

$$3x^3 - 30x^2 + 75x = 3x(x^2 - 10x + 25)$$

Now factor the trinomial. Find factors of 25 that sum to –10, or use the formula for a perfect square trinomial, $A^2 - 2AB + B^2 = (A-B)^2$.

$$\begin{aligned} 3x^3 - 30x^2 + 75x &= 3x(x^2 - 10x + 25) \\ &= 3x(x^2 - 2 \cdot x \cdot 5 + 5^2) \\ &= 3x(x-5)^2 \end{aligned}$$

7b. Factor $x^2 - 36a^2 + 20x + 100$.

Regroup factors and look for opportunities to factor within groupings.

$$x^2 - 36a^2 + 20x + 100 = (x^2 + 20x + 100) - 36a^2$$

Factor the expression in parentheses using $A^2 + 2AB + B^2 = (A+B)^2$.

$$\begin{aligned} (x^2 + 20x + 100) - 36a^2 &= (x^2 + 2 \cdot x \cdot 10 + 10^2) - 36a^2 \\ &= (x+10)^2 - 36a^2 \end{aligned}$$

✎ *Pencil Problem #7* ✎

7a. Factor $20y^4 - 45y^2$.

7b. Factor $x^2 - 12x + 36 - 49y^2$.

This last form is the difference of squares. Factor using $A^2 - B^2 = (A+B)(A-B)$.

$$\begin{aligned}(x+10)^2 - 36a^2 &= (x+10)^2 - (6a)^2\\ &= [(x+10)+6a][(x+10)-6a]\\ &= (x+10+6a)(x+10-6a)\end{aligned}$$

So, $x^2 - 36a^2 + 20x + 100 = (x+10+6a)(x+10-6a)$.

Objective #8: Factor algebraic expressions containing fractional and negative exponents.

✔ *Solved Problem #8*

8. Factor and simplify: $x(x-1)^{-\frac{1}{2}} + (x-1)^{\frac{1}{2}}$.

The GCF is $(x - 1)$ with the smaller exponent. Thus, the GCF is $(x-1)^{-\frac{1}{2}}$.

$$\begin{aligned}x(x-1)^{-\frac{1}{2}} + (x-1)^{\frac{1}{2}} &= (x-1)^{-\frac{1}{2}} \cdot x + (x-1)^{-\frac{1}{2}} \cdot (x-1)\\ &= (x-1)^{-\frac{1}{2}}[x+(x-1)]\\ &= (x-1)^{-\frac{1}{2}}(2x-1)\\ &= \frac{2x-1}{(x-1)^{1/2}}\end{aligned}$$

✎ *Pencil Problem #8* ✎

8. Factor and simplify: $(x+3)^{\frac{1}{2}} - (x+3)^{\frac{3}{2}}$.

<u>**Answers**</u> for Pencil Problems <u>*(Textbook Exercise references in parentheses)*</u>:

1a. $3x(x+2)$ *(P.5 #3)* **1b.** $(x+5)(x+3)$ *(P.5 #7)* **2.** $(x-2)(x^2+5)$ *(P.5 #11)*
3a. $(x-5)(x-3)$ *(P.5 #21)* **3b.** $(3x-2)(3x-1)$ *(P.5 #31)*
4. $(3x-5y)(3x+5y)$ *(P.5 #43)* **5a.** $(x+1)^2$ *(P.5 #49)* **5b.** $(3x-1)^2$ *(P.5 #55)*
6a. $(x+3)(x^2-3x+9)$ *(P.5 #57)* **6b.** $(2x-1)(4x^2+2x+1)$ *(P.5 #61)*
7a. $5y^2(2y+3)(2y-3)$ *(P.5 #83)* **7b.** $(x-6+7y)(x-6-7y)$ *(P.5 #85)* **8.** $-(x+3)^{\frac{1}{2}}(x+2)$ *(P.5 #97)*

Section P.6
Rational Expressions

Ouch! That Hurts!!

Though it may not be fun to get a flu shot, it is a great way to protect yourself from getting sick!

In this section of the textbook, one of the application problems will explore the costs for inoculating various percentages of the population.

Objective #1: Specify numbers that must be excluded from the domain of a rational expression.

✔ *Solved Problem #1*

1. Find all real numbers that must be excluded from the domain of each rational expression.

1a. $\dfrac{7}{x+5}$

The denominator, $x + 5$, would be 0 if $x = -5$. We must exclude -5 from the domain.

1b. $\dfrac{7x}{x^2-5x-14}$

Factor the denominator.

$x^2-5x-14=(x-7)(x+2)$

The first factor would be 0 if $x = 7$. The second factor would be 0 if $x = -2$. We must exclude -2 and 7 from the domain.

✎ *Pencil Problem #1* ✐

1. Find all real numbers that must be excluded from the domain of each rational expression.

1a. $\dfrac{7}{x-3}$

1b. $\dfrac{x+5}{x^2-25}$

Objective #2: Simplify rational expressions.

✔ Solved Problem #2

2a. Simplify $\frac{x^3+3x^2}{x+3}$.

Note that $x \neq -3$ since -3 would make the denominator 0.

Factor the numerator and divide out common factors.

$$\frac{x^3+3x^2}{x+3} = \frac{x^2(x+3)}{x+3} = \frac{x^2\cancel{(x+3)}}{\cancel{x+3}}$$
$$= x^2,\ x \neq -3$$

2b. Simplify $\frac{x^2-1}{x^2+2x+1}$.

By factoring the denominator, $x^2+2x+1=(x+1)^2$, we see that $x \neq -1$.

Factor the numerator and denominator and divide out common factors.

$$\frac{x^2-1}{x^2+2x+1} = \frac{(x+1)(x-1)}{(x+1)(x+1)} = \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(x+1)}$$
$$= \frac{x-1}{x+1},\ x \neq -1$$

Pencil Problem #2

2a. Simplify $\frac{3x-9}{x^2-6x+9}$.

2b. Simplify $\frac{y^2+7y-18}{y^2-3y+2}$.

Objective #3: Multiply rational expressions.

✔ Solved Problem #3

3. Multiply: $\frac{x+3}{x^2-4} \cdot \frac{x^2-x-6}{x^2+6x+9}$.

Factor and divide by common factors.

$$\frac{x+3}{x^2-4} \cdot \frac{x^2-x-6}{x^2+6x+9} = \frac{x+3}{(x+2)(x-2)} \cdot \frac{(x-3)(x+2)}{(x+3)(x+3)}$$
$$= \frac{\cancel{x+3}}{\cancel{(x+2)}(x-2)} \cdot \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+3)}(x+3)}$$
$$= \frac{x-3}{(x-2)(x+3)},\ x \neq -3, -2, 2$$

To see which values must be excluded from the domain, look at the factored forms of the denominators in the second step.

Pencil Problem #3

3. Multiply: $\frac{x^2-5x+6}{x^2-2x-3} \cdot \frac{x^2-1}{x^2-4}$.

Objective #4: Divide rational expressions.

✔ *Solved Problem #4*

4. Divide: $\frac{x^2-2x+1}{x^3+x} \div \frac{x^2+x-2}{3x^2+3}$.

Invert the divisor and multiply.

$$\frac{x^2-2x+1}{x^3+x} \div \frac{x^2+x-2}{3x^2+3} = \frac{x^2-2x+1}{x^3+x} \cdot \frac{3x^2+3}{x^2+x-2}$$

$$= \frac{(x-1)\cancel{(x-1)}}{x\cancel{(x^2+1)}} \cdot \frac{3\cancel{(x^2+1)}}{(x+2)\cancel{(x-1)}}$$

$$= \frac{3(x-1)}{x(x+2)},\ x \neq -2, 0, 1$$

✎ *Pencil Problem #4* ✎

4. Divide: $\frac{x^2-25}{2x-2} \div \frac{x^2+10x+25}{x^2+4x-5}$.

Objective #5: Add and subtract rational expressions.

✔ *Solved Problem #5*

5a. Subtract: $\frac{x}{x+1} - \frac{3x+2}{x+1}$.

The expressions have the same denominator. Subtract numerators.

$$\frac{x}{x+1} - \frac{3x+2}{x+1} = \frac{x-(3x+2)}{x+1}$$

$$= \frac{x-3x-2}{x+1}$$

$$= \frac{-2x-2}{x+1}$$

$$= \frac{-2\cancel{(x+1)}}{\cancel{x+1}}$$

$$= -2,\ x \neq -1$$

✎ *Pencil Problem #5* ✎

5a. Add: $\frac{4x+1}{6x+5} + \frac{8x+9}{6x+5}$.

5b. Add: $\dfrac{3}{x+1}+\dfrac{5}{x-1}$.

The denominators are not equal and have no common factor. Use the property $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{ad+bc}{bd}$.

$$\begin{aligned}\frac{3}{x+1}+\frac{5}{x-1}&=\frac{3(x-1)+5(x+1)}{(x+1)(x-1)}\\&=\frac{3x-3+5x+5}{(x+1)(x-1)}\\&=\frac{8x+2}{(x+1)(x-1)}\\&=\frac{2(4x+1)}{(x+1)(x-1)},\ x\neq -1,1\end{aligned}$$

5b. Add: $\dfrac{3}{x+4}+\dfrac{6}{x+5}$.

5c. Subtract: $\dfrac{x}{x^2-10x+25}-\dfrac{x-4}{2x-10}$.

Factor the denominators.

$x^2-10x+25=(x-5)(x-5)$

$2x-10=2(x-5)$

LCD = $2(x-5)(x-5)$

$$\begin{aligned}&\frac{x}{x^2-10x+25}-\frac{x-4}{2x-10}\\&=\frac{x}{(x-5)(x-5)}-\frac{x-4}{2(x-5)}\\&=\frac{2x}{2(x-5)(x-5)}-\frac{(x-4)(x-5)}{2(x-5)(x-5)}\\&=\frac{2x-(x-4)(x-5)}{2(x-5)^2}\\&=\frac{2x-(x^2-9x+20)}{2(x-5)^2}\\&=\frac{2x-x^2+9x-20}{2(x-5)^2}\\&=\frac{-x^2+11x-20}{2(x-5)^2},\ x\neq 5\end{aligned}$$

5c. Subtract: $\dfrac{3x}{x^2+3x-10}-\dfrac{2x}{x^2+x-6}$.

Objective #6: Simplify complex rational expressions.

✔ Solved Problem #6

6a. Simplify: $\dfrac{\dfrac{1}{x}-\dfrac{3}{2}}{\dfrac{1}{x}+\dfrac{3}{4}}$.

Subtract and add in the numerator and denominator to obtain a single rational expression in each.

$$\frac{1}{x}-\frac{3}{2}=\frac{1\cdot 2}{x\cdot 2}-\frac{3\cdot x}{2\cdot x}=\frac{2}{2x}-\frac{3x}{2x}=\frac{2-3x}{2x}$$
$$\frac{1}{x}+\frac{3}{4}=\frac{1\cdot 4}{x\cdot 4}+\frac{3\cdot x}{4\cdot x}=\frac{4}{4x}+\frac{3x}{4x}=\frac{4+3x}{4x}$$

Now return to the original complex fraction.

$$\begin{aligned}\frac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}}&=\frac{\frac{2-3x}{2x}}{\frac{4+3x}{4x}}\\&=\frac{2-3x}{2x}\cdot\frac{4x}{4+3x}\\&=\frac{2-3x}{\cancel{2x}}\cdot\frac{2\cdot\cancel{2x}}{4+3x}\\&=\frac{2(2-3x)}{4+3x},\ x\neq 0,-\frac{4}{3}\end{aligned}$$

✎ Pencil Problem #6 ✎

6a. Simplify: $\dfrac{1+\dfrac{1}{x}}{3-\dfrac{1}{x}}$.

6b. Simplify: $\dfrac{\dfrac{1}{x+7}-\dfrac{1}{x}}{7}$.

The LCD of the fractions within the complex fraction is $x(x+7)$. Multiply the numerator and the denominator of the complex fraction by the LCD.

$$\begin{aligned}\frac{\frac{1}{x+7}-\frac{1}{x}}{7}&=\frac{\left(\frac{1}{x+7}-\frac{1}{x}\right)x(x+7)}{7x(x+7)}\\&=\frac{\frac{1}{x+7}\cdot x(x+7)-\frac{1}{x}\cdot x(x+7)}{7x(x+7)}\\&=\frac{x-(x+7)}{7x(x+7)}\\&=\frac{-7}{7x(x+7)}\\&=\frac{-1}{x(x+7)},\ x\neq -7,0\end{aligned}$$

6b. Simplify: $\dfrac{\dfrac{3}{x-2}-\dfrac{4}{x+2}}{\dfrac{7}{x^2-4}}$.

<u>Answers</u> <u>for Pencil Problems</u> <u>*(Textbook Exercise references in parentheses)*</u>:

1a. 3 *(P.6 #1)* **1b.** $-5, 5$ *(P.6 #3)* **2a.** $\frac{3}{x-3},\ x \neq 3$ *(P.6 #7)* **2b.** $\frac{y+9}{y-1},\ y \neq 1, 2$ *(P.6 #11)*

3. $\frac{x-1}{x+2},\ x \neq -2, -1, 2, 3$ *(P.6 #19)* **4.** $\frac{x-5}{2},\ x \neq -5, 1$ *(P.6 #29)*

5a. $2,\ x \neq -\frac{5}{6}$ *(P.6 #33)* **5b.** $\frac{9x+39}{(x+4)(x+5)},\ x \neq -5, -4$ *(P.6 #41)*

5c. $\frac{x^2-x}{(x+5)(x-2)(x+3)},\ x \neq -5, -3, 2$ *(P.6 #53)*

6a. $\frac{x+1}{3x-1},\ x \neq 0, \frac{1}{3}$ *(P.6 #61)* **6b.** $-\frac{x-14}{7},\ x \neq -2, 2$ *(P.6 #67)*