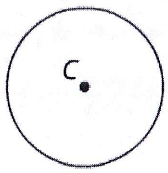
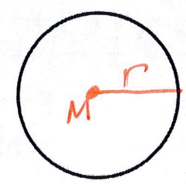
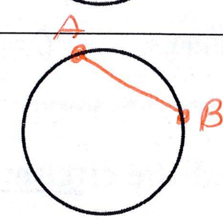
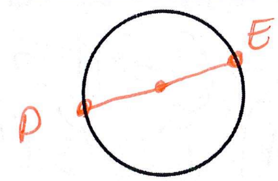
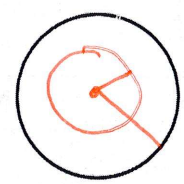
10.1 NOTES – Circles and Circumferences

A <u>Circle</u> is the locus or set of points in a plane equidistant from a given point called the <u>Center</u> of the circle. A circle is named by its center point.




Circle $C$ or $\odot C$

$\odot C$

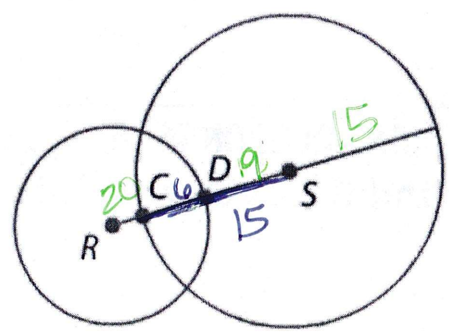
| | |
|---|---|
| A <u>radius</u> is a segment with endpoints at the center and on the circle |   |
| A <u>chord</u> is a segment with endpoints on the circle |   |
| A <u>Diameter</u> of a circle is the cord that passes through the center of the circle. |   |
| <u>Concentric Circles</u> are coplanar circles that have the same center |  |

**The diameter of $\odot S$ is 30 units, the diameter of $\odot R$ is 20 units, and $DS = 9$ units. Find $CD$.**

$CD + DS = 15$

$CD + 9 = 15$

$CD = 6$



10.1 NOTES – Circles and Circumferences

What is Pi?

3.141592654 $C = \pi d$ $\pi = \frac{C}{d}$

**KeyConcept Radius and Diameter Relationships**

If a circle has radius $r$ and diameter $d$, the following relationships are true.

**Radius Formula** $r = \frac{d}{2}$ or $r = \frac{1}{2}d$ **Diameter Formula** $d = 2r$

**KeyConcept Circumference**

**Words** If a circle has diameter $d$ or radius $r$, the circumference $C$ equals the diameter times pi or twice the radius times pi.

**Symbols** $C = \pi d$ or $C = 2\pi r$

**Find the circumference of each circle described. Round to the nearest hundredth.**

**4A.** radius = 2.5 centimeters

$C = \pi 5$

$C = 5\pi$

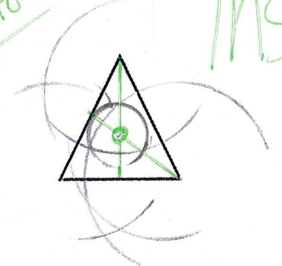
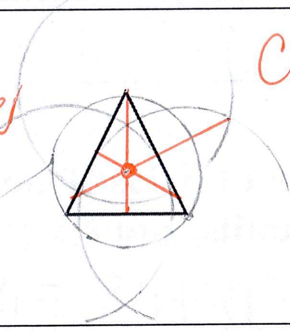
$C = 15.71$

$\boxed{\begin{array}{l} C = 106.4 \\ 106.4 = \pi d \\ 33.87 = d \end{array}}$

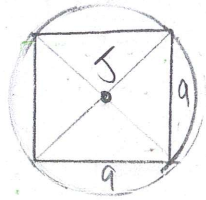
**4B.** diameter = 16 feet

$C = 16\pi$

$C = 50.27$

| INSCRIBED | CIRCUMSCRIBED |
|---|---|
| Find the Incenter angle bisectors  inside | outside Find per Bisectors to find Circumcenter  Circumcenter |

**SHORT RESPONSE** **A square with side length of 9 inches is inscribed in ⊙*J*. Find the exact circumference of ⊙*J*.**




$\boxed{\text{diameter } 9\sqrt{2}}$

Inside Diameter

$9^2 + 9^2 = d^2$

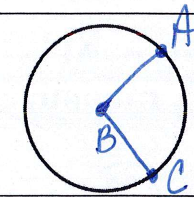
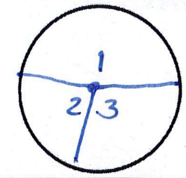
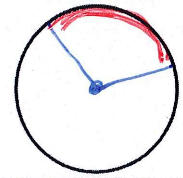
$81 + 81 = d^2$

$\sqrt{162} = \sqrt{d^2}$

$\sqrt{162} = d$

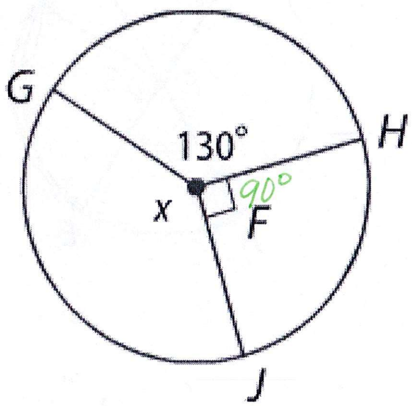
$\sqrt{81} \cdot \sqrt{2} = d$

10.2 NOTES – Angles and Arcs 10-2

| | |
|---|---|
| A Central Angle of a circle is an angle with the vertex in the center of the circle<br>$\angle ABC$ is a center angle of $\odot B$ |  |
| The Sum of the central angles in a circle is always 360°<br>$m\angle 1 + m\angle 2 + m\angle 3 = 360°$ |  |
| An Arc length is the portion of a circle defined by two endpoints. A central angle separates 2 arcs. |  |

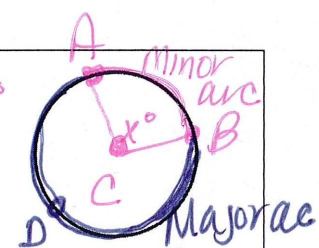
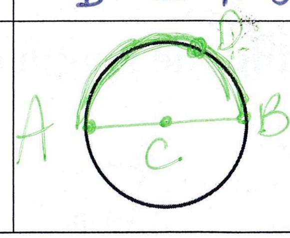
$\frac{l}{Cir} = \frac{x}{360}$ Arc length $l = \frac{x}{360} \cdot 2\pi r$

Find the value of x



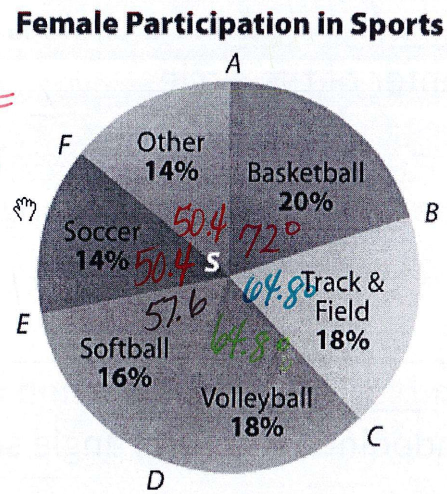

$360 - 130 - 90 = x$

$x = 140°$

| | |
|---|---|
| A Minor arc is the shortest arc connecting two endpoints on a circle<br>$m\overset{\frown}{AB} = m\angle ACB = x°$<br><br>A Major arc the longest one $m\overset{\frown}{ADB} = 360 - m\overset{\frown}{AB} = 360 - x$ |  |
| A Semicircle is an arc with endpoints that lie on a diameter<br>$m\overset{\frown}{ADB} = 180°$ |  |

10.2 NOTES – Angles and Arcs

**SPORTS Refer to the circle graph. Find each measure.**




$360 \cdot .2 =$

$360 \cdot .18 =$

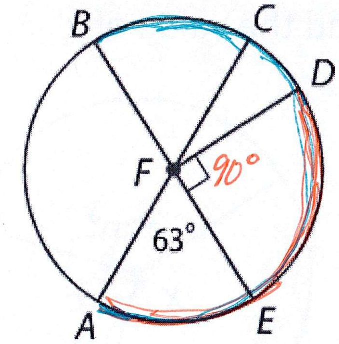
**a.** $m\widehat{CD}$ $64.8°$

$360 \cdot .14$

$360 \cdot .16$

**3A.** $m\widehat{EF}$ $50.4°$

**Find each measure in ⊙F.**

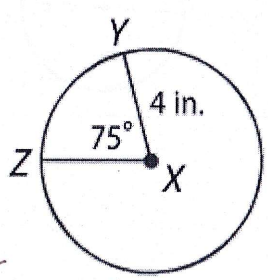



**a.** $m\widehat{AED}$ $63 + 90 = 153°$

**b.** $m\widehat{ADB}$ $180 + 63 = 243°$

**Find the length of $\widehat{ZY}$. Round to the nearest hundredth.**

**a.**



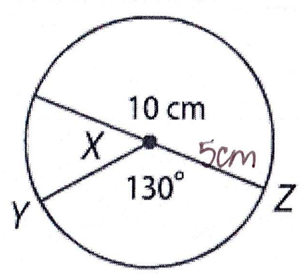

$l = \frac{X}{360} \cdot 2\pi r$

$l = \frac{75}{360} \cdot 2\pi(4)$

$\frac{75\pi}{45} \approx \frac{5\pi}{3}$

$\boxed{\widehat{ZY} = 5.24}$ $\frac{8\pi}{360} \cdot 75$ $\frac{\pi}{45} \cdot 75$

**b.**




$l = \frac{130}{360} \cdot 2\pi(5)$

$\frac{\cancel{5}10 \cdot \pi \cdot 130}{360}$ 48

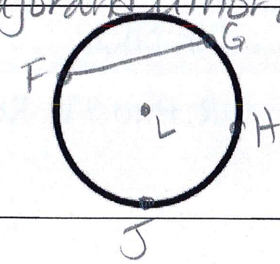
$\frac{5 \cdot 13\pi}{18}$ $\frac{65\pi}{18}$

$\boxed{YZ = 11.34cm}$

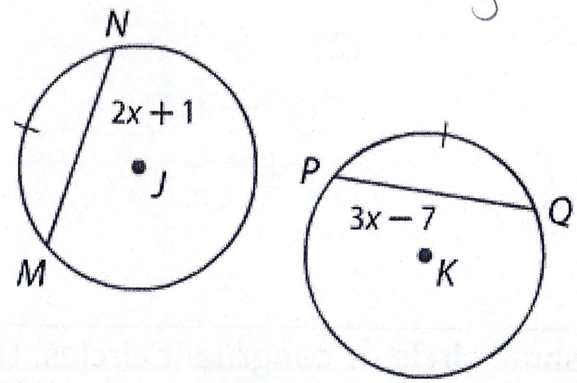
* Chord is a segment w/ endpoints on a Circle.

If a chord is not a diameter, then its endpoints divide the circle into a major and minor arc

## 10.3 NOTES –Arcs and Chords

| | |
|---|---|
| In the same circle or congruent circles two minor arcs are Congruent **IFF** their corresponding Chords are Congruent<br>$\overset{\frown}{FG} \cong \overset{\frown}{HJ}$ iff $\overline{FG} = \overline{HJ}$ |  |

**ALGEBRA** In the figures, $\odot J \cong \odot K$ and $\overset{\frown}{MN} \cong \overset{\frown}{PQ}$. Find $PQ$.



Minor arcs are Congruent

$\overline{MN} = \overline{PQ}$

$2x+1 = 3x-7$

$-2x+7 \quad -2x+7$

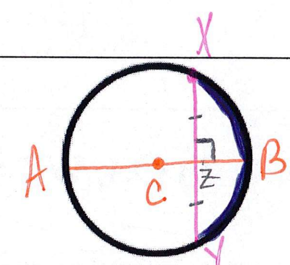
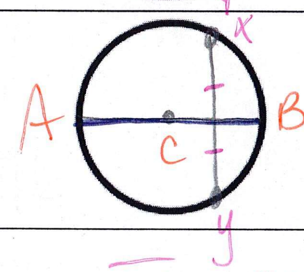
$\boxed{8 = x}$

$PQ = 3(8)-7$

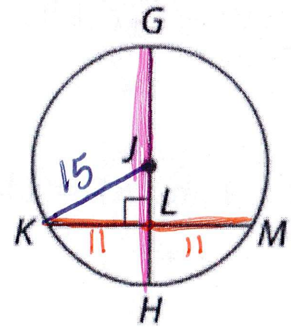
$PQ = 24-7$

$PQ = 17$

$\overline{PQ} = 17$

| | |
|---|---|
| If the diameter (or radius) is Perpendicular to a chord, then it bisects the chord and its arch<br>$\overline{XZ} \cong \overline{YZ}$ and $\overset{\frown}{XB} \cong \overset{\frown}{BY}$ |  |
| The perp. Bisector of a chord is a diameter of a circle<br>$\overline{XY}$<br>If AB is a perpendicular bisector of chord XY the AB is a diameter of $\odot C$ |  |

**STAINED GLASS** In the stained glass window, diameter $\overline{GH}$ is 30 inches long and chord $\overline{KM}$ is 22 inches long. Find $JL$.



$\overline{GH} = 30$ inches

JK is radius

$r = 15$

Draw a $\triangle$

$11^2 + JL^2 = 15^2$

$121 + (JL)^2 = 225$

$-121 \qquad -121$

$(JL)^2 = 104$

$JL = \sqrt{104}$

$\sqrt{4}\sqrt{26}$

$2\sqrt{26} \simeq 10.20$ inches long

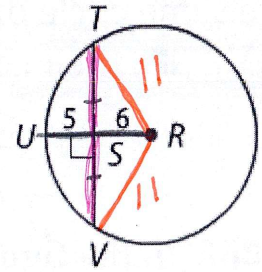
Exact

10.3 NOTES –Arcs and Chords

## GuidedPractice

**4.** In ⊙$R$, find $TV$. Round to the nearest hundredth.

$\overline{UR}$ radius




$11^2 = 6^2 + TS^2$

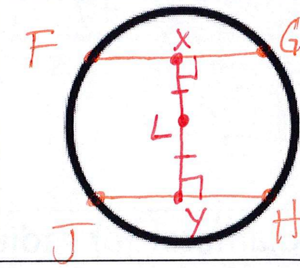
$121 = 36 + TS^2$

$85 = TS^2$

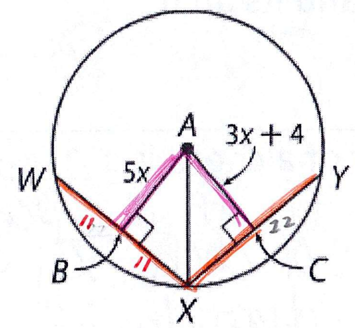
$\sqrt{85} = TS$

$\sqrt{85} + \sqrt{85} = TV$

$\boxed{TV = 18.44}$

| | |
|---|---|
| In the same circle or congruent circles, two chords are congruent **IFF** they are equidistant from the center<br><br>$\overline{FG} \cong \overline{JH}$ iff $LX = LY$ |  |

**ALGEBRA** In ⊙$A$, $WX = XY = 22$. Find $AB$.




$\overline{WX} \cong \overline{XY}$

then

$\overline{AB} \cong \overline{AC}$

$5x = 3x + 4$

$-3x \quad -3x$

$2x = 4$

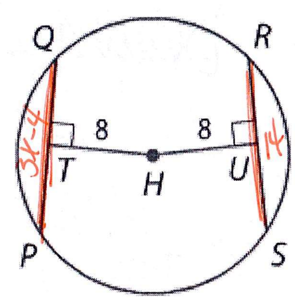
$x = 2$

$AB = 5 \cdot 2 = 10$

$\boxed{AB = 10}$

## GuidedPractice

**5.** In ⊙$H$, $PQ = 3x - 4$ and $RS = 14$. Find $x$.




$TH = HU \therefore$

$PQ = RS$

$3x - 4 = 14$

$+4 \quad +4$

$3x = 18$

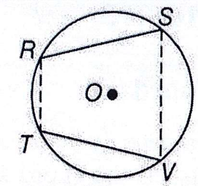
$x = 6$

$\boxed{x = 6}$

NAME ______________________ DATE ______________ PERIOD __________

# 10-3 Study Guide and Intervention
## *Arcs and Chords*

**Arcs and Chords** Points on a circle determine both chords and arcs. Several properties are related to points on a circle. In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



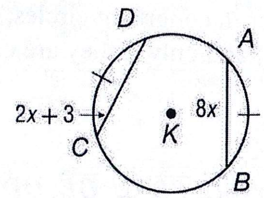
$\widehat{RS} \cong \widehat{TV}$ if and only if $\overline{RS} \cong \overline{TV}$.

**Example: In ⊙$K$, $\widehat{AB} \cong \widehat{CD}$. Find $AB$.**

$\widehat{AB}$ and $\widehat{CD}$ are congruent arcs, so the corresponding chords $\overline{AB}$ and $\overline{CD}$ are congruent.

| | |
|---|---|
| $AB = CD$ | Definition of congruent segments |
| $8x = 2x + 3$ | Substitution |
| $x = \frac{1}{2}$ | Simplify. |

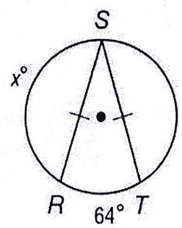
So, $AB = 8\left(\frac{1}{2}\right)$ or 4.




**Exercises**

**ALGEBRA Find the value of $x$ in each circle.**

**1.**



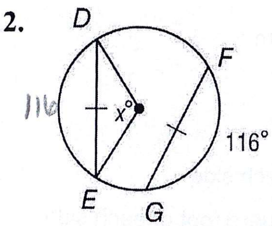

$x°$

$2x+64=360$

$2x=296$

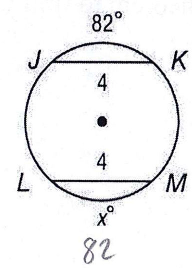
$\boxed{x=148°}$

**2.**



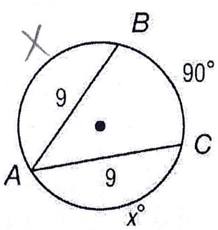

$x=116°$

**3.**




$x=82°$

**4.**

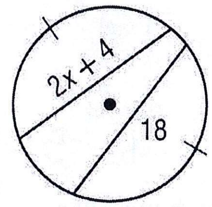



$2x+90=360$

$2x=270$

$x=135°$

**5.**



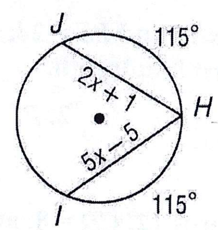

$2x+4=18$

$-4\ \ -4$

$2x=14$

$\boxed{x=7}$

**6.**



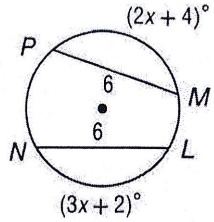

$2x+1=5x-5$

$-2x+5\ \ -2x+5$

$6=3x$

$\boxed{2=x}$

**7.**

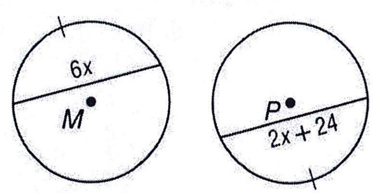



$2x+4=3x+2$

$-2x-2\ \ -2x-2$

$\boxed{2=x}$

**8.** ⊙$M$ ≅ ⊙$P$



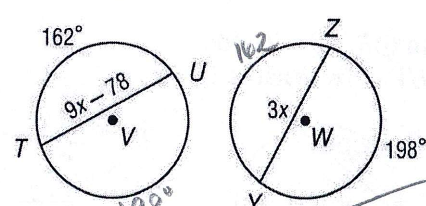

$2x+24=6x$

$-2x\ \ \ \ -2x$

$24=4x$

$\boxed{6=x}$

**9.** ⊙$V$ ≅ ⊙$W$




$\boxed{x=13}$

$9x-78=3x$

$-9x\ \ \ \ -9x$

$-78=-6x$

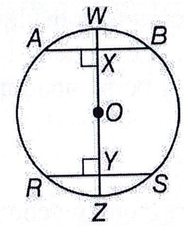
NAME ______________________ DATE ______________ PERIOD ________

# 10-3 Study Guide and Intervention *(continued)*

## *Arcs and Chords*

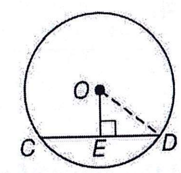
### Diameters and Chords

- In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle, the perpendicular bisector of a chord is the diameter (or radius).
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



If $\overline{WZ} \perp \overline{AB}$, then $\overline{AX} \cong \overline{XB}$ and $\widehat{AW} \cong \widehat{WB}$.
If $OX = OY$, then $\overline{AB} \cong \overline{RS}$.
If $\overline{AB} \cong \overline{RS}$, then $\overline{AB}$ and $\overline{RS}$ are equidistant from point $O$.

**Example: In $\odot O$, $\overline{CD} \perp \overline{OE}$, $OD = 15$, and $CD = 24$. Find $OE$.**



A diameter or radius perpendicular to a chord bisects the chord, so $ED$ is half of $CD$.

$ED = \frac{1}{2}(24)$

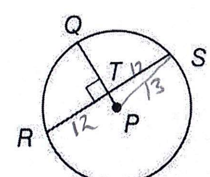
$= 12$

Use the Pythagorean Theorem to find $x$ in $\triangle OED$.

| | |
|---|---|
| $(OE)^2 + (ED)^2 = (OD)^2$ | Pythagorean Theorem |
| $(OE)^2 + 12^2 = 15^2$ | Substitution |
| $(OE)^2 + 144 = 225$ | Simplify. |
| $(OE)^2 = 81$ | Subtract 144 from each side. |
| $OE = 9$ | Take the positive square root of each side. |

### Exercises

**In $\odot P$, the radius is 13 and $RS = 24$. Find each measure. Round to the nearest hundredth.**



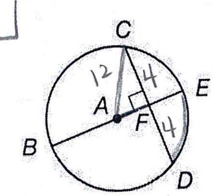
$r = 13$

**1.** $RT$ $\boxed{12}$

**2.** $PT$ $13^2 = 12^2 + PT^2$
$169 = 144 + PT^2$
$25 = PT^2$
$\boxed{5 = PT}$

**3.** $TQ$ $13 - 5 = \boxed{8}$

**In $\odot A$, the diameter is 12, $CD = 8$, and $m\widehat{CD} = 90$. Find each measure. Round to the nearest hundredth.**



$d = 12$

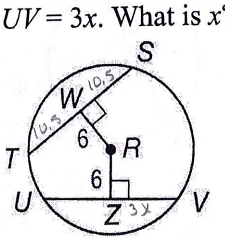
$90°$ $\boxed{14.47 = AF}$

**4.** $m\widehat{DE}$ $45°$

**5.** $FD$ $4$

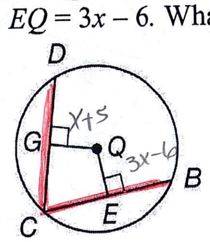
**6.** $AF$ $6^2 = 4^2 + AF^2$
$36 \neq 16 + AF^2$
$AF^2 = 20$
$AF = \sqrt{4}\sqrt{5}$
$\boxed{2\sqrt{5}}$

**7.** In $\odot R$, $TS = 21$ and $UV = 3x$. What is $x$?



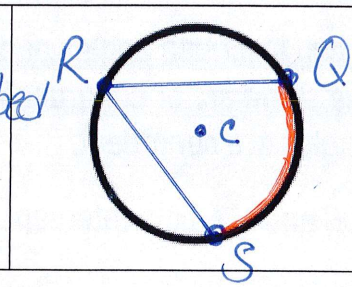
$21 = 3x$
$\boxed{x = 7}$

**8.** In $\odot Q$, $\overline{CD} \cong \overline{CB}$, $GQ = x + 5$ and $EQ = 3x - 6$. What is $x$?

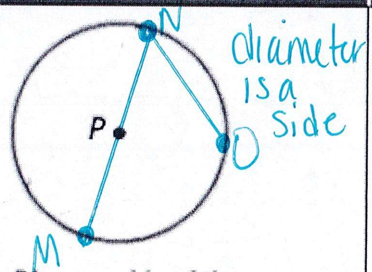
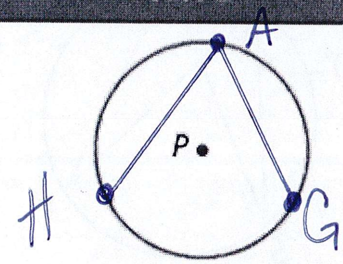
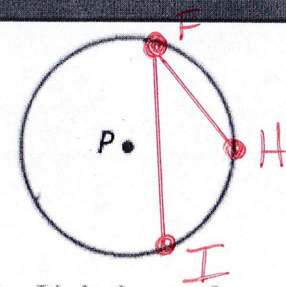


$x + 5 = 3x - 6$
$-x + 6 \quad -x \quad +6$
$11 = 2x$
$\boxed{x = 5.5}$

## 10.4 Inscribed Angles

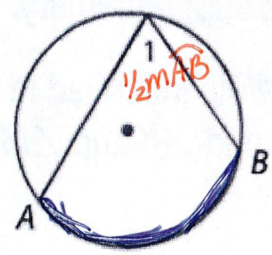
| | |
|---|---|
| An Inscribed Angle has a vertex on a circle and sides that contain chords of the circle. ⊙C ∠QRS is inscribed<br><br>An Intercepted Arc has endpoints on the sides of an inscribed angle. |  |

There are three ways that an angle can be inscribed in a circle

| Case 1 | Case 2 | Case 3 |
|---|---|---|
|  diameter is a side |  |  |
| Center *P* is on a side of the inscribed angle. | Center *P* is inside the inscribed angle. | The center *P* is in the exterior of the inscribed angle. |

In Case 1, the side of the angle is a diameter of the circle.

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.



$m\angle 1 = \underline{\frac{1}{2} m\widehat{AB}}$ $m\widehat{AB} = \underline{2m\angle 1}$

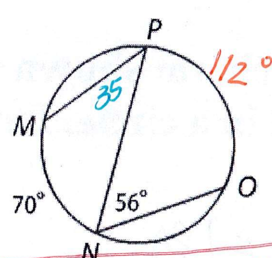
**Find each measure.**

**a.** $m\angle P$

$35^\circ$

**b.** $m\widehat{PO}$

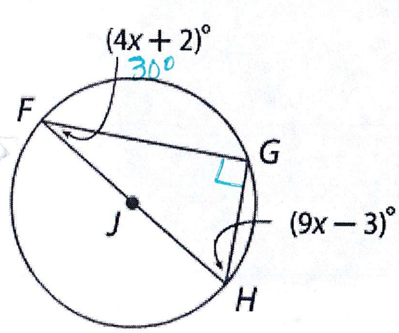
$112^\circ$



**ALGEBRA Find $m\angle F$.**

Semicircle

$\boxed{\text{An inscribed angle of a } \triangle \text{ intercepts a diameter or semi circle iff the angle is right}}$ *



$4x+2+9x-3=90$

$13x-1=90$

$+1 \quad +1$

$13x=91$

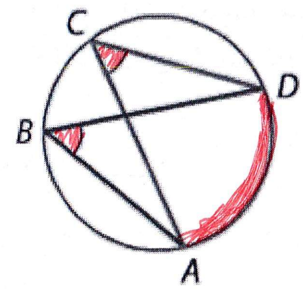
$x=7$

$m\angle F=30^\circ$

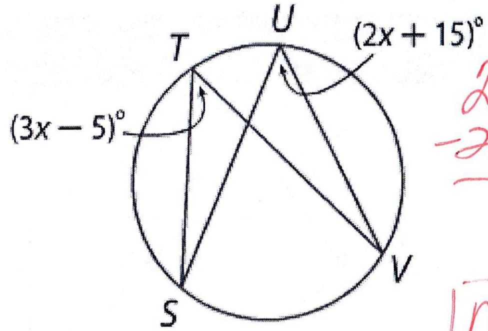
# 10.4 Inscribed Angles

If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.

$\angle B$ and $\angle C$ both intercept $\widehat{AD}$. So, $\angle B \cong$ $\angle C$




**ALGEBRA** Find $m\angle T$.

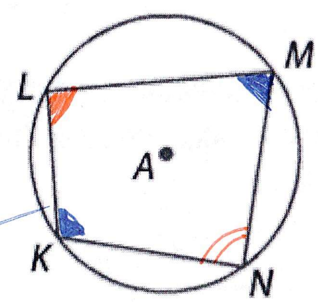



$2x + 15 = 3x - 5$
$-2x \quad +5 \quad -2x + 5$
$20 = x$

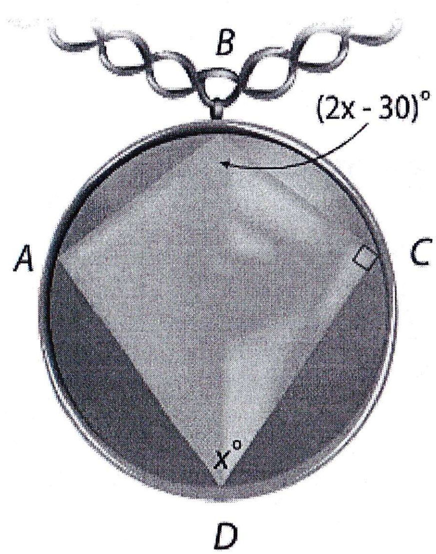
$\boxed{m\angle T = 55^\circ}$

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

If quadrilateral $KLMN$ is inscribed in $\odot A$, then $\angle L$ and $\angle N$ are Supplementary and $\angle K$ and $\angle M$ are Supplementary



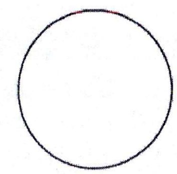
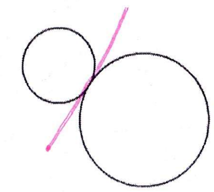
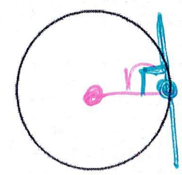

**JEWELRY** The necklace charm shown uses a quadrilateral inscribed in a circle. Find $m\angle A$ and $m\angle B$.




$2x - 30 + x = 180$
$3x = 210$
$x = 70$

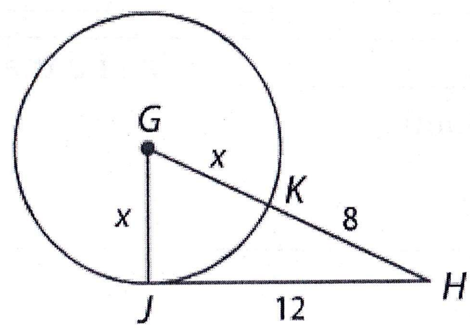
$m\angle A = 70^\circ$
$m\angle B = 110^\circ$

# 10.5 and 10.6 Tangents Secants and Angles

| | |
|---|---|
| A Tangent is a line in the same plane as a circle that intersects the circle at exactly one point called the Point of tangency |  |
| A tangent that touches two circles at once is a Common tangent<br><br>DRAW IN THE COMMON TANGENTS -----> |  |
| A tangent is Perpendicular to the radius of a circle |  |

## Example 3 Use a Tangent to Find Missing Values

$\overline{JH}$ is tangent to $\odot G$ at $J$. Find the value of $x$.

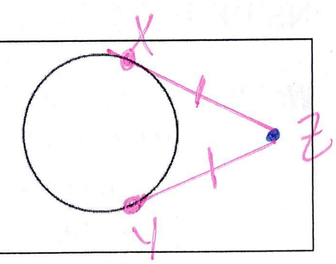



$(x+8)^2 = x^2 + 12^2$

$x^2 + 16x + 64 = x^2 + 144$

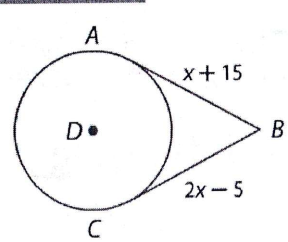
$16x = 80$

$\boxed{x = 5}$

| | |
|---|---|
| If two segments from the same exterior point are tangent to a circle, then they are Congruent $XZ = YZ$ |  |

## Example 4 Use Congruent Tangents to Find Measures

**ALGEBRA** $\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$. Find the value of $x$.




$x + 15 = 2x - 5$

$-x \qquad -x$

$15 = x - 5$

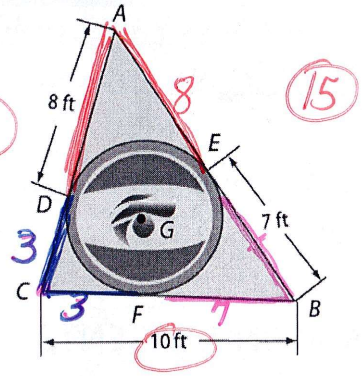
$20 = x$

# 10.5 and 10.6 Tangents Secants and Angles

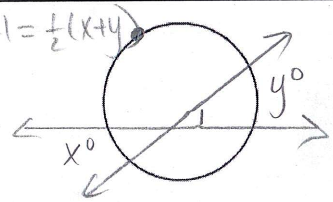
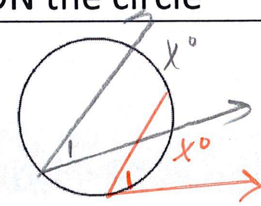
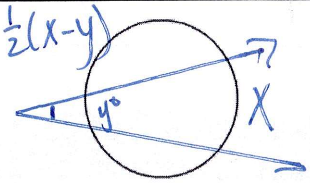
**Real-World Example 5 Find Measures in Circumscribed Polygons**

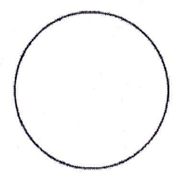
**GRAPHIC DESIGN** A graphic designer is giving directions to create a larger version of the triangular logo shown. If $\triangle ABC$ is circumscribed about $\odot G$, find the perimeter of $\triangle ABC$.

P = 36 feet



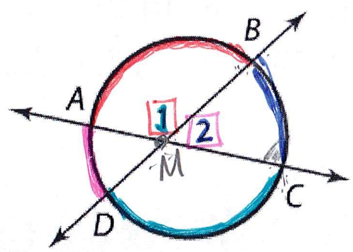

XXXXXXXXXXX-10.6-XXXXXXXXXXXXXXX

| There are three cases for when two lines can intersect with a circle: | | |
|---|---|---|
| INSIDE the circle | ON the circle | OUTSIDE the circle |
| $m\angle 1 = \frac{1}{2}(x+y)$  | $m\angle 1 = \frac{1}{2}x$  | $m\angle 1 = \frac{1}{2}(x-y)$  |

| | |
|---|---|
| A Secant is a line that intersects a circle in exactly two points |  |

## INSIDE THE CIRCLE

**Words** If two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the *sum* of the measure of the arcs intercepted by the angle and its vertical angle.



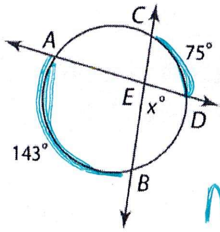

**Example** $m\angle 1 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$ and $m\angle 2 = \frac{1}{2}(m\widehat{DA} + m\widehat{BC})$

In your own words: ① $\overleftrightarrow{AC}$ and $\overleftrightarrow{DB}$ Intersect at M ① Given
② $m\angle 1 = m\angle MBC + m\angle MCB$ ② Exterior angle thm
③ $m\angle MBC = \frac{1}{2} m\widehat{DC}$ $m\angle MCB = \frac{1}{2} m\widehat{AB}$ ③ the measured of an inscribed ∠ equals half the measure of intercepted arc
④ $m\angle 1 = \frac{1}{2} m\widehat{DC} + \frac{1}{2} m\widehat{AB}$ ④ Substit
⑤ $m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$ Distributive

## 10.5 and 10.6 Tangents Secants and Angles

Inside $m\angle 1 = \frac{1}{2}(x+y)$

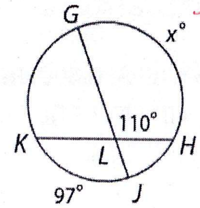
Example 1:




$m\angle AEB = \frac{1}{2}(143 + 75)$

$m\angle AEB = 109°$

Example 2:




123°

$110 = \frac{1}{2}(x + 97)$

$220 = x + 97$

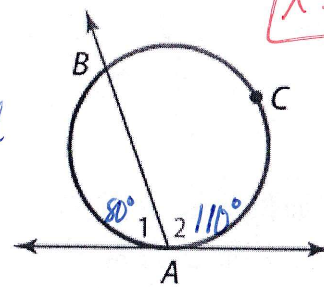
$\boxed{x = 123}$

ON THE CIRCLE

**Words** If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

on circle

**Example** $m\angle 1 = \frac{1}{2} m\widehat{AB}$ and $m\angle 2 = \frac{1}{2} m\widehat{ACB}$



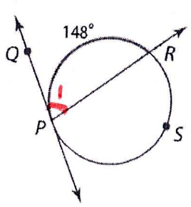

In your own words:

$m\angle 1 = \frac{1}{2}(\widehat{AB})$

$80 = \frac{1}{2}(AB)$

$\boxed{160 = AB}$

Example 1:



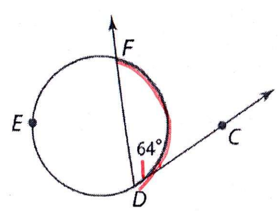

$m\angle QPR =$

$m\angle 1 = \frac{1}{2}(PR)$

$m\angle 1 = \frac{1}{2}(148)$

$\boxed{m\angle 1 = 74°}$

Example 2:




$m\widehat{DEF} =$

$\frac{1}{2} m\widehat{FD} = m\angle 1$

$\frac{1}{2} m\widehat{FD} = 64°$

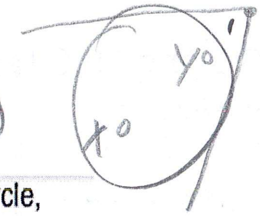
$m\widehat{FD} = 128°$

$360 - 128 = m\widehat{DEF}$

$\boxed{= 232}$

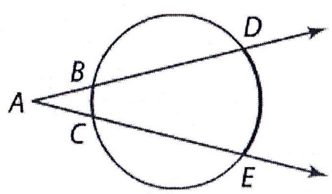
# 10.5 and 10.6 Tangents Secants and Angles
## OUTSIDE THE CIRCLE

$m\angle 1 = \frac{1}{2}(x - y)$



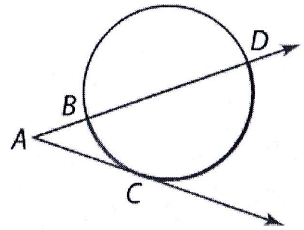

**Words** If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

**Examples**



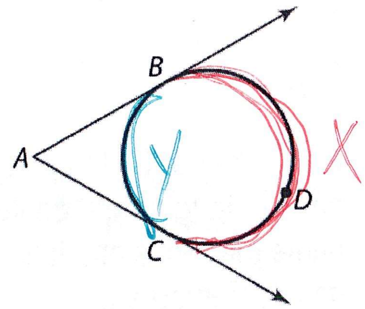

**Two Secants**

$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$




**Secant-Tangent**

$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$

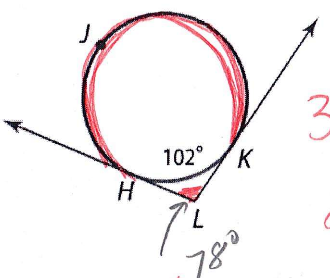



**Two Tangents**

$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

In your own words:

Example 1:



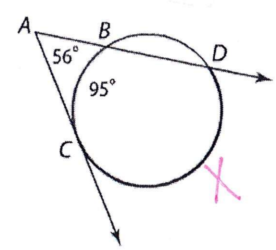

360 - 102

258

$m\angle L = \frac{1}{2}(258 - 102)$

$m\angle L = 129 - 51 = 78°$

Example: 2



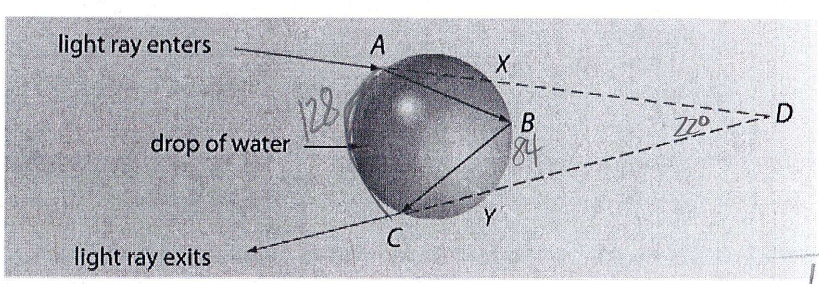

$56 = \frac{1}{2}(x - 95)$

$112 = x - 95$

$207 = x$

$\boxed{m\widehat{CD} = 207}$

**SCIENCE** **The diagram shows the path of a light ray as it hits a drop of water. The ray is bent, or *refracted*, at points *A*, *B*, and *C*. If $m\widehat{AC} = 128$ and $m\widehat{XBY} = 84$, what is $m\angle D$?**




$m\angle D = \frac{1}{2}(128 - 84)$

$m\angle D = \frac{1}{2}(44)$

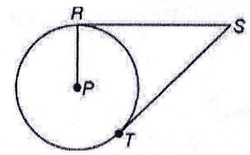
$\boxed{m\angle D = 22°}$

NAME ______________________ DATE ______________ PERIOD ________

# 10-5 Study Guide and Intervention
## *Tangents*

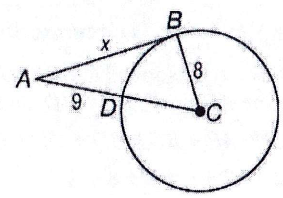
**Tangents** A **tangent** to a circle intersects the circle in exactly one point, called the **point of tangency**. There are important relationships involving tangents. A **common tangent** is a line, ray, or segment that is tangent to two circles in the same plane.

- A line is tangent to a circle if and only if it is perpendicular to a radius at a point of tangency.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.




If $\overline{RS} \perp \overline{RP}$, then $\overline{SR}$ is tangent to $\odot P$. If $\overline{SR}$ is tangent to $\odot P$, then $\overline{RS} \perp \overline{RP}$.
If $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

**Example: $\overline{AB}$ is tangent to $\odot C$. Find $x$.**



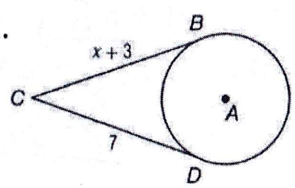

$AB$ is tangent to $\odot C$, so $\overline{AB}$ is perpendicular to radius $\overline{BC}$. $\overline{CD}$ is a radius, so $CD = 8$ and $AC = 9 + 8$ or 17. Use the Pythagorean Theorem with right $\triangle ABC$.

| | |
|---|---|
| $(AB)^2 + (BC)^2 = (AC)^2$ | Pythagorean Theorem |
| $x^2 + 8^2 = 17^2$ | Substitution |
| $x^2 + 64 = 289$ | Simplify. |
| $x^2 = 225$ | Subtract 64 from each side. |
| $x = 15$ | Take the positive square root of each side. |

### Exercises

**Find $x$. Assume that segments that appear to be tangent are tangent.**

**1.**

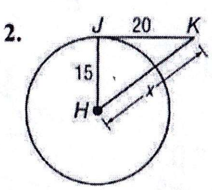



4

$X+3=7$

$\boxed{X=4}$

**2.**



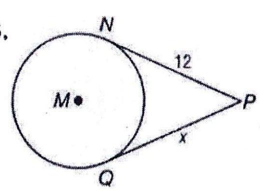

25

$15^2+20^2=(15+X)^2$

$625=(15+X)^2$

$\boxed{25=X}$

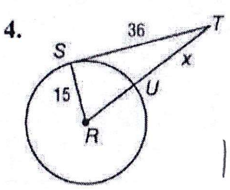
**3.**




12

$\boxed{X=12}$

**4.**




24

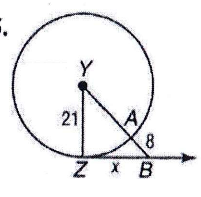
$15^2+36^2=(15+U)^2$

$1521=(15+U)^2$

$39=15+U$

$\boxed{24=U}$

**5.**




20

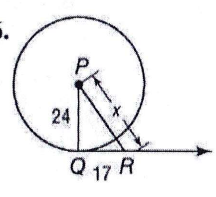
$X^2+21^2=29^2$

$X^2+441=841$

$X^2=400$

$\boxed{X=20}$

**6.**




$\sqrt{865} \approx 29.41$

$X^2=17^2+24^2$

$289+576$

$X^2=865$

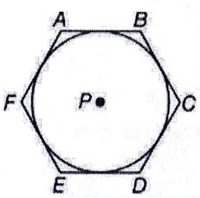
$X=\sqrt{865} \approx 29.41$

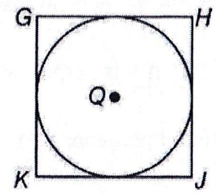
NAME ______________________ DATE ______________ PERIOD ________

# 10-5 Study Guide and Intervention *(continued)*

## *Tangents*

**Circumscribed Polygons** When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.

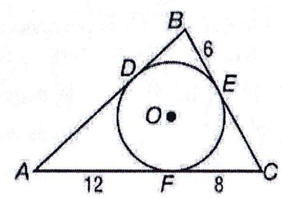



Hexagon *ABCDEF* is circumscribed about $\odot P$. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{FA}$ are tangent to $\odot P$.

Square *GHJK* is circumscribed about $\odot Q$. $\overline{GH}$, $\overline{JH}$, $\overline{JK}$, and $\overline{KG}$ are tangent to $\odot Q$.

**Example: $\triangle ABC$ is circumscribed about $\odot O$. Find the perimeter of $\triangle ABC$.**




$\triangle ABC$ is circumscribed about $\odot O$, so points *D*, *E*, and *F* are points of tangency. Therefore $AD = AF$, $BE = BD$, and $CF = CE$.

$P = AD + AF + BE + BD + CF + CE$

$= 12 + 12 + 6 + 6 + 8 + 8$

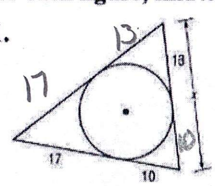
$= 52$

The perimeter is 52 units.

### Exercises

**For each figure, find *x*. Then find the perimeter.**

1.

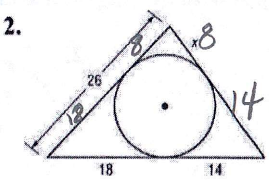



$\boxed{23}$ ; $\boxed{80}$ = P

X = 23

P = 23 + 27 + 30

2.



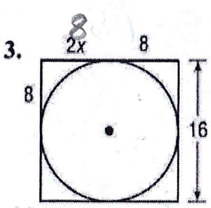

14 8 ; 80

$\boxed{X = 8}$

P = 22 + 32 + 26

$\boxed{P = 80}$

3.

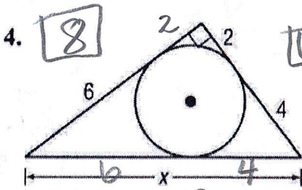



$\boxed{X = 4}$

4 ; 64

4(16) = $\boxed{64 = P}$

4. $\boxed{8}$



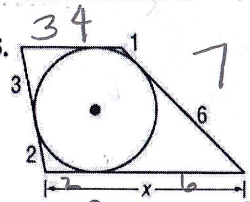

$\boxed{16}$

10 ; 24

$\boxed{X = 10}$

$\boxed{P = 24}$

5.




5

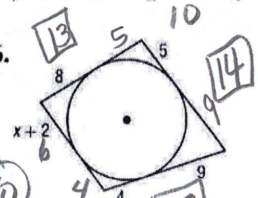
7

8 ; 24

$\boxed{X = 8}$

$\boxed{P = 24}$

6. $\boxed{13}$




$\boxed{14}$

6 ; 52

10

$\boxed{13}$

$\boxed{X = 6}$

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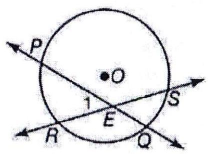
# 10-6 Study Guide and Intervention

## *Secants, Tangents, and Angle Measures*

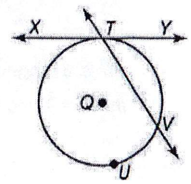
**Intersections On or Inside a Circle** A line that intersects a circle in exactly two points is called a **secant**. The measures of angles formed by secants and tangents are related to intercepted arcs.

- If two secants or chords intersect in the interior of a circle, then the measure of the angle formed is one half the sum of the measure of the arcs intercepted by the angle and its vertical angle.




$m\angle 1 = \frac{1}{2}(m\widehat{PR} + m\widehat{QS})$

- If a secant (or chord) and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

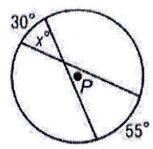



$m\angle XTV = \frac{1}{2}m\widehat{TUV}$

$m\angle YTV = \frac{1}{2}m\widehat{TV}$

**Example 1: Find $x$.**

The two chords intersect inside the circle, so $x$ is equal to one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.



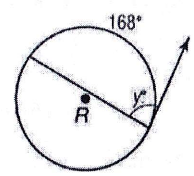

$x = \frac{1}{2}(30 + 55)$

$= \frac{1}{2}(85)$

$= 42.5$

**Example 2: Find $y$.**

The chord and the tangent intersect at the point of tangency, so the measure of the angle is one half the measure of its intercepted arc.



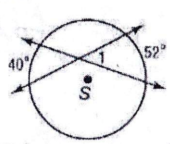

$y = \frac{1}{2}(168)$

$= 84$

**Exercises**

**Find each measure. Assume that segments that appear to be tangent are tangent.**

**1.** $m\angle 1$

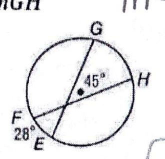



46

$m\angle 1 = \frac{1}{2}(40+52)$

$\boxed{m\angle 1 = 46°}$

**2.** $m\widehat{GH}$



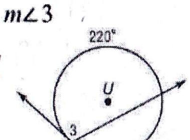

$m\angle$ $45 = \frac{1}{2}(28+x)$

62 $90 = 28 + x$

$m\angle GH$

$\boxed{x = 62 \quad m\widehat{GH} = 62}$

**3.** $m\angle 3$

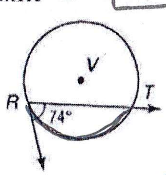



110

$m\angle 3 = \frac{1}{2}(220)$

$\boxed{m\angle 3 = 110°}$

**4.** $m\widehat{RT}$



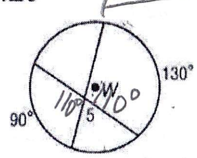

148

$m\widehat{RT} =$

$74 = \frac{1}{2}m\widehat{RT}$

$\boxed{148 = m\widehat{RT}}$

**5.** $m\angle 5$

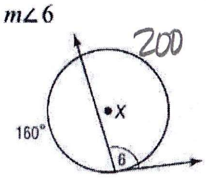



70

$m = \frac{1}{2}(130+90)$

$\boxed{m\angle 5 = 70°}$

**6.** $m\angle 6$




100

$m\angle 6 = \frac{1}{2}(200)$

$\boxed{m\angle 6 = 100°}$

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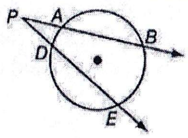
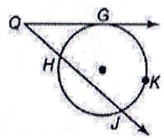
# 10-6 Study Guide and Intervention *(continued)*
## *Secants, Tangents, and Angle Measures*

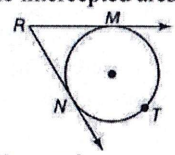
**Intersections Outside a Circle** If secants and tangents intersect outside a circle, they form an angle whose measure is related to the intercepted arcs. If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.



$\overrightarrow{PB}$ and $\overrightarrow{PE}$ are secants.
$m\angle P = \frac{1}{2}(m\widehat{BE} - m\widehat{AD})$



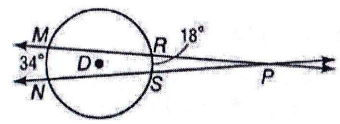
$\overrightarrow{QG}$ is a tangent. $\overrightarrow{QJ}$ is a secant.
$m\angle Q = \frac{1}{2}(m\widehat{GKJ} - m\widehat{GH})$



$\overrightarrow{RM}$ and $\overrightarrow{RN}$ are tangents.
$m\angle R = \frac{1}{2}(m\widehat{MTN} - m\widehat{MN})$

**Example: Find $m\angle MPN$.**

$\angle MPN$ is formed by two secants that intersect in the exterior of a circle.



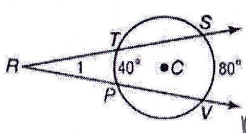
$$m\angle MPN = \frac{1}{2}(m\widehat{MN} - m\widehat{RS})$$
$$= \frac{1}{2}(34 - 18)$$
$$= \frac{1}{2}(16) \text{ or } 8$$

The measure of the angle is 8.

**Exercises**

**Find each measure. Assume that segments that appear to be tangent are tangent.**

**1.** $m\angle 1$

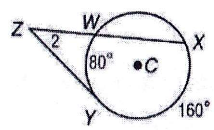


20

$m\angle 1 = \frac{1}{2}(80-40)$

$\boxed{m\angle 1 = 20°}$

**2.** $m\angle 2$



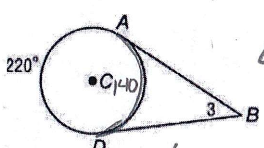
40

$m\angle 2 = \frac{1}{2}(160-80)$

$m\angle 2 = \frac{1}{2}(80)$

$\boxed{m\angle 2 = 40°}$

**3.** $m\angle 3$

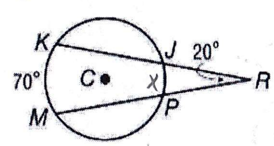


40

$\boxed{m\angle 3 = 40°}$

$m\angle 3 = \frac{1}{2}(220-140)$

**4.** $m\widehat{JP}$



30

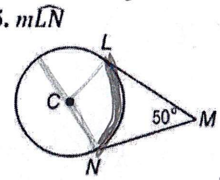
$20° = \frac{1}{2}(70-x)$

$40 = 70 - x$

$30 = x$

$\boxed{m\widehat{JP} = 30}$

**5.** $m\widehat{LN}$



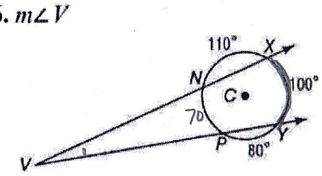
130

$50 = \frac{1}{2}(x-y)$

$100 = (x-y)$

$\boxed{130° = m\widehat{LN}}$

**6.** $m\angle V$



15

$m\angle V = \frac{1}{2}(100-70)$

$\boxed{m\angle V = 15°}$

NAME ______________________ DATE ______________ PERIOD ________

# 10-7 Study Guide and Intervention *(continued)*

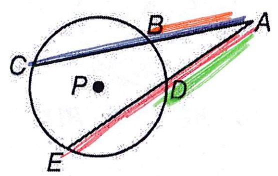
## *Special Segments in a Circle*

**Segments Intersecting Outside a Circle** If secants and tangents intersect outside a circle, then two products are equal. A **secant segment** is a segment of a secant line that has exactly one endpoint on the circle. A secant segment that lies in the exterior of the circle is called an **external secant segment**. A **tangent segment** is a segment of a tangent with one endpoint on the circle.

Part 2

- If two secants are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

Secants



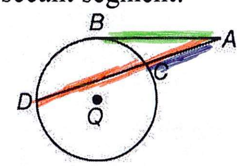

$\overline{AC}$ and $\overline{AE}$ are secant segments.

$\overline{AB}$ and $\overline{AD}$ are external secant segments.

$AC \cdot AB = AE \cdot AD$

Part 3

- If a tangent segment and a secant segment are drawn to a circle from exterior point, then the square of the measure of the tangent segment is equal to the product of the measures the secant segment and its external secant segment.




$\overline{AB}$ is a tangent segment.

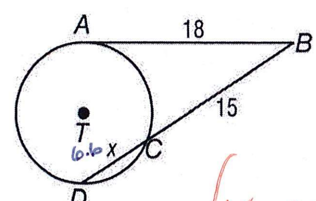
$\overline{AD}$ is a secant segment.

$\overline{AC}$ is an external secant segment.

$(AB)^2 = AD \cdot AC$

**Example: $\overline{AB}$ is tangent to the circle. Find $x$. Round to the nearest tenth.**

The tangent segment is $\overline{AB}$, the secant segment is $BD$, and the external secant segment is $\overline{BC}$.




$(AB)^2 = BC \cdot BD$

| | |
|---|---|
| $(18)^2 = 15(15 + x)$ | Substitution. |
| $324 = 225 + 15x$ | Multiply. |
| $99 = 15x$ | Subtract 225 from both sides. |
| $6.6 = x$ | Divide both sides by 15. |

$18^2 = (x+15)(15)$

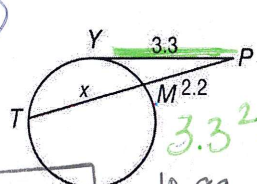
$324 = 15x + 225$

$99 = 15x$ $\boxed{x = 6.6}$

**Exercises**

**Find $x$. Round to the nearest tenth. Assume segments that appear to be tangent are tangent.**

**1.**




2.8

$3.3^2 = (2.2 + x)2.2$

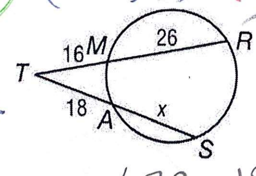
$10.89 = 4.84 + 2.2x$

$6.05 = 2.2x$

$\boxed{x = 2.8}$

**2.**

$(16+26)(16) = (x+18)18$

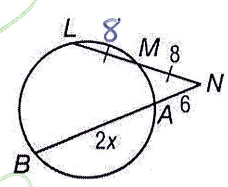



$\boxed{19.3}$

$672 = 18x + 324$

$348 = 18x$ $\boxed{x = 19.3}$

**3.**




7.7

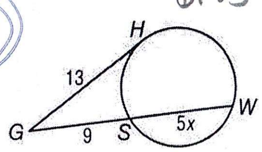
$(16)8 = (6 + 2x)6$

$128 = 36 + 12x$

$92 = 12x$

$\boxed{x = 7.7}$

**4.**




2.0

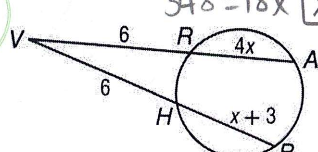
$13^2 = (5x + 9)9$

$169 = 45x + 81$

$88 = 45x$ $\boxed{2.0}$

$x = 1.96$

**5.**



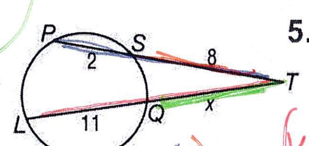

1.0

$(6 + 4x)6 = (6 + x + 3) \cdot 6$

$36 + 24x = 6x + 54$

$18x = 18$ $\boxed{x = 1}$

**6.**




5.0

$\boxed{x = 5}$

$(10)(8) = (x + 11)x$

$80 = x^2 + 11x$

$0 = x^2 + 11x - 80$

$-80$

$16, -5$

$0 = (x+16)(x-5)$

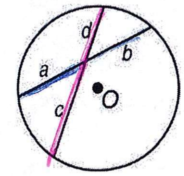
$\boxed{x = 5}$

NAME Key DATE PERIOD

# 10-7 Study Guide and Intervention

## *Special Segments in a Circle*

**Segments Intersecting Inside a Circle** If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

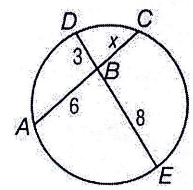



$a \cdot b = c \cdot d$

Part 1

**Example: Find** $x$.

The two chords intersect inside the circle, so the products $AB \cdot BC$ and $EB \cdot BD$ are equal.




$AB \cdot BC = EB \cdot BD$

| | |
|---|---|
| $6 \cdot x = 8 \cdot 3$ | Substitution |
| $6x = 24$ | Multiply. |
| $x = 4$ | Divide each side by 6. |

$6x = 24$

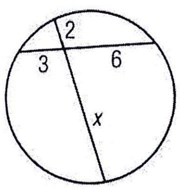
$\boxed{x = 4}$

$AB \cdot BC = EB \cdot BD$

### Exercises

**Find** $x$. **Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.**

**1.**

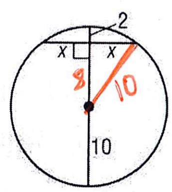



9

$2x = 18$

$\boxed{x = 9}$

**2.**



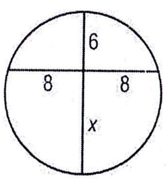

6

$x^2 = 2(18)$

$x^2 = 36$

$\boxed{x = 6}$

**3.**

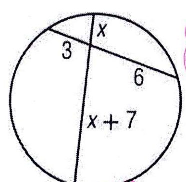



10.7

$6x = 64$

$\boxed{x = 10.7}$

**4.**




2

$-18$

$9, -2 \mid 7$

$x(x+7) = 18$

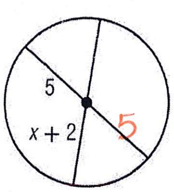
$x^2 + 7x - 18 = 0$

$(x+9)(x-2) = 0$

$x = -9$ no

$x = 2$

**5.**




3

$25 = (x+2)(x+2)$

$25 = x^2 + 4x + 4$

$0 = x^2 + 4x - 21$

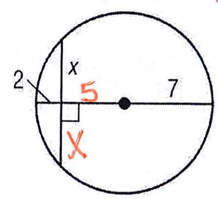
$-21$

$7, -3 \mid 4$

$(x+7)(x-3)$

$x \neq -7$ $\quad x = 3$

**6.**




4.9

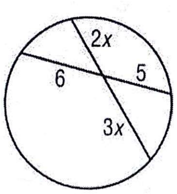
$x^2 = 2(12)$

$x^2 = 24$

$x = \pm\sqrt{4}\sqrt{6}$

$\boxed{2\sqrt{6} \quad x = 4.9}$

**7.**




2.2

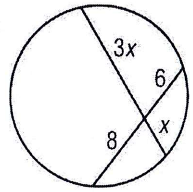
$2x \cdot 3x^2 = 5 \cdot 6$

$6x^2 = 30$

$x^2 = 5$

$\boxed{x = \sqrt{5} \quad x = 2.2}$

**8.**




4

$x(3x) = 48$

$3x^2 = 48$

$x^2 = 16$

$\boxed{x = 4}$