

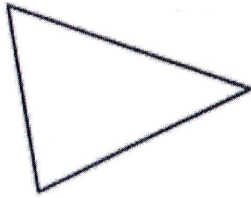
Key

Honors Math 2

Chapter 4 Notes

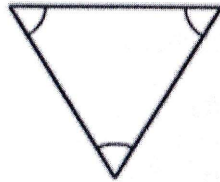
4.1/4.2

Classifying triangles by their angles:



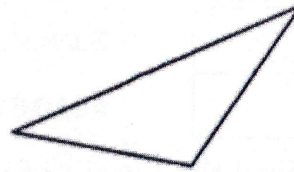
3 acute angles

Acute Triangle



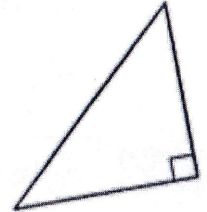
3 congruent acute angles

Equiangular



1 obtuse angle

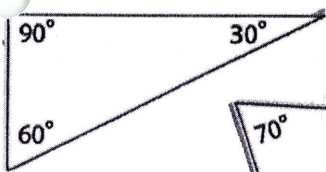
Obtuse Δ



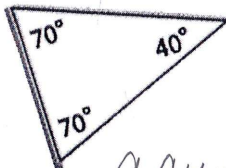
1 right angle

Right Δ

Classify each angle by its angles using the best description:

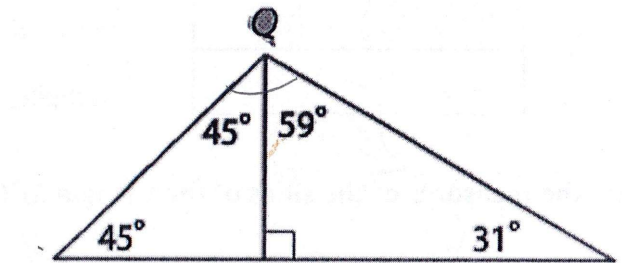


Right Δ



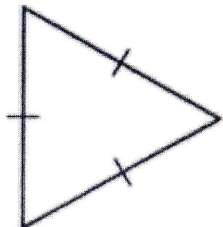
acute Δ

Classify ΔPQR as specifically as possible:



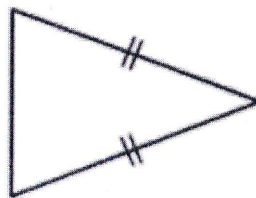
Obtuse Δ because $45^\circ + 59^\circ > 90^\circ$

We can also classify triangles based on their sides:



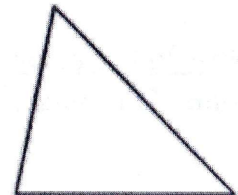
3 congruent sides

Equilateral



at least 2 congruent sides

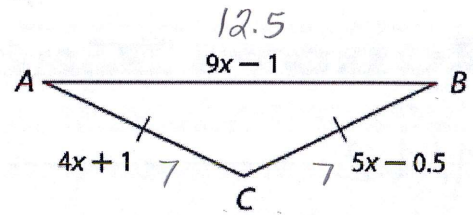
Isosceles



no congruent sides

Scalene

ALGEBRA Find the measures of the sides of isosceles triangle ABC.



Step 1 Find x .

$$AC = CB$$

Given

| |
|-------------------------------|
| $4x + 1 = 5x - 0.5$ |
| $\quad -4x \quad \quad -4x$ |
| $1 = 1x - 0.5$ |
| $\quad +0.5 \quad \quad +0.5$ |
| $1.5 = 1x$ |

Substitution

Subtract $4x$ from each side.

Add 0.5 to each side.

$x = 1.5$

Step 2 Substitute to find the length of each side.

$$AC = 4x + 1$$

Given

| |
|--------------|
| $4(1.5) + 1$ |
| $6 + 1$ |

$$x = 1.5$$

Given

$$= 7$$

$$AC = 7$$

$$AB = 9x - 1$$

Given

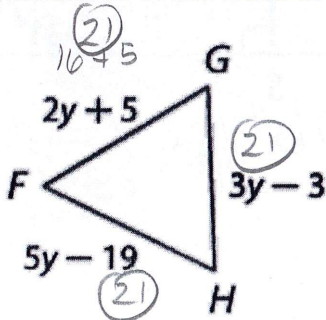
| |
|--------------|
| $9(1.5) - 1$ |
| $13.5 - 1$ |

$$x = 1.5$$

Simplify.

12.5

Find the measures of the sides of the triangle $\triangle FGH$:



Equilateral

$$\begin{array}{r} 2y + 5 = 3y - 3 \\ -2y \quad -2y \\ \hline \end{array}$$

$$\begin{array}{r} 5 = y - 3 \\ +3 \quad +3 \\ \hline \end{array}$$

$$y = 8$$

The **Triangle Angle-Sum Theorem** says that

'The sum of the measures of the angles is 180° '

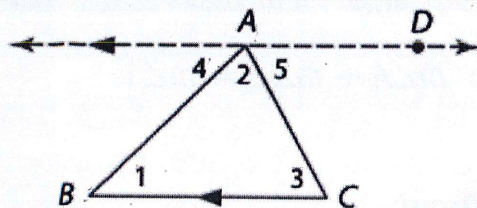
An **Auxiliary Line** is an extra line or segment drawn in a figure to help analyze geometric relationships. As with any statement in a proof, you must justify any properties of an auxiliary line that you have drawn.

PROOF: Triangle Angle-Sum Theorem

Given: $\triangle ABC$

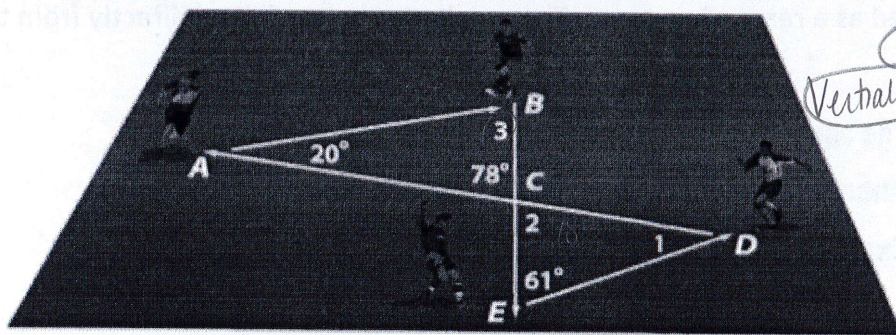
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$

Proof:



| Statements | Reasons |
|---|--|
| 1. $\triangle ABC$ | 1. Given |
| 2. Draw \overleftrightarrow{AD} through A parallel to \overline{BC} . | 2. Parallel Postulate |
| 3. $\angle 4$ and $\angle BAD$ form a linear pair. | 3. Def. of a linear pair |
| 4. $\angle 4$ and $\angle BAD$ are Supp | 4. if 2 \angle 's form linear pair they are Supp |
| 5. $m\angle 4 + m\angle BAD = 180$ | 5. def of Supp \angle 's |
| 6. $m\angle BAD = m\angle 2 + m\angle 5$ | 6. Angle Addition Postulate |
| 7. $m\angle 2 + m\angle 5 + m\angle 4 = 180$ | 7. Substitution |
| 8. $\angle 4 \cong \angle 1, \angle 5 \cong \angle 3$ | 8. alternate Interior \angle 's |
| 9. $m\angle 4 = m\angle 1, m\angle 5 = m\angle 3$ | 9. Congruent \angle 's def |
| 10. $m\angle 1 + m\angle 2 + m\angle 3 = 180$ | 10. Substitution |

Soccer The diagram shows the path of the ball in a passing drill created by four friends. Find the measure of each numbered angle.



$m\angle 3 = 82^\circ$
 Vertical $m\angle 2 = 78^\circ$
 $m\angle 1 = 180 - 61 - 78$
 $m\angle 1 = 41^\circ$

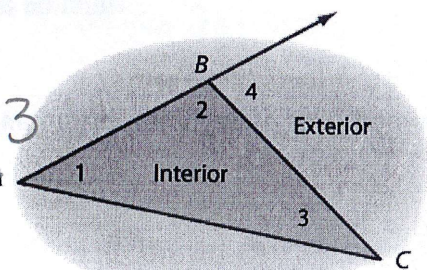
$180 - 20 - 78 =$
 $100 - 78 = 82$

Exterior Angles Theorem:

each exterior angle of a triangle is equal to the sum of the two remote interior angles

$m\angle 4 = m\angle 1 + m\angle 3$

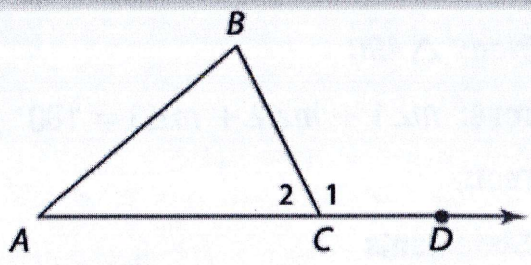
always



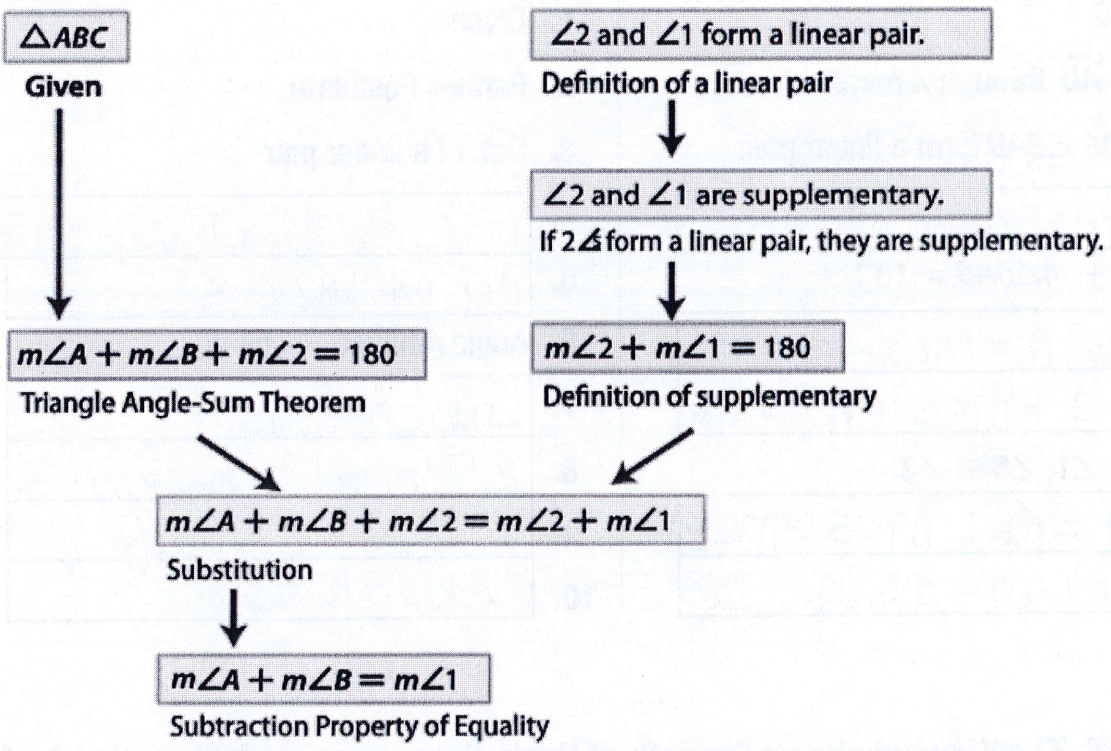
Proof Exterior Angle Theorem Flow proof: uses boxes and arrows.

Given: $\triangle ABC$ *Triangle add up 2 180*

Prove: $m\angle A + m\angle B = m\angle 1$



Flow Proof:

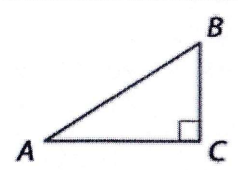


A corollary is a theorem with a proof that follows as a direct result of another theorem. As with theorems, corollaries can be used as a reason in a proof. The corollaries below follow directly from the Triangle Angle-Sum Theorem.

4.1 The acute angles of a right triangle are complementary.

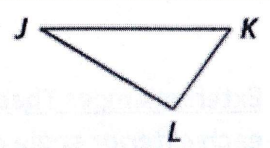
Abbreviation: *Acute \angle s of a rt. \triangle are comp.*

Example: If $\angle C$ is a right angle, then $\angle A$ and $\angle B$ are complementary.



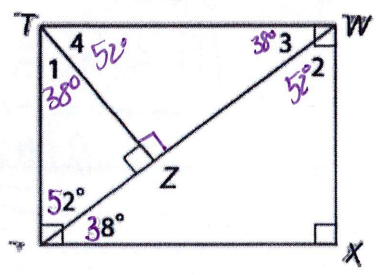
4.2 There can be at most one right or obtuse angle in a triangle.

Example: If $\angle L$ is a right or an obtuse angle, then $\angle J$ and $\angle K$ must be acute angles.



Find the measures of each numbered angle:

- $m\angle 1 = 38^\circ$
- $m\angle 2 = 52^\circ$
- $m\angle 3 = 38^\circ$
- $m\angle 4 = 52^\circ$



Honors Math 2

Chapter 4 Notes

4.3

Geometric figures that are *exactly* the same shape and size are congruent.

| Congruent | Not Congruent |
|---|--|
| | |
| <p>While positioned differently, Figures 1, 2, and 3 are exactly the same shape and size.</p> | <p>Figures 4 and 5 are exactly the same shape but not the same size. Figures 5 and 6 are the same size but not exactly the same shape.</p> |

The corresponding parts of congruent shapes are also congruent:

Words

Two polygons are congruent if and only if their corresponding parts are congruent.

Example

Corresponding Angles

$$\angle A \cong \angle H \quad \angle B \cong \angle J \quad \angle C \cong \angle K$$

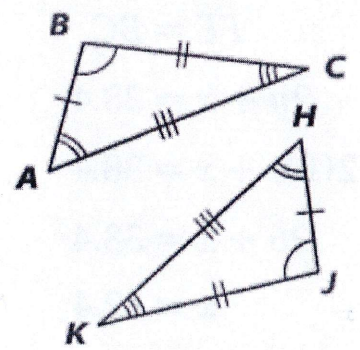
Corresponding Sides

$$\overline{AB} \cong \overline{HJ} \quad \overline{BC} \cong \overline{JK} \quad \overline{AC} \cong \overline{HK}$$

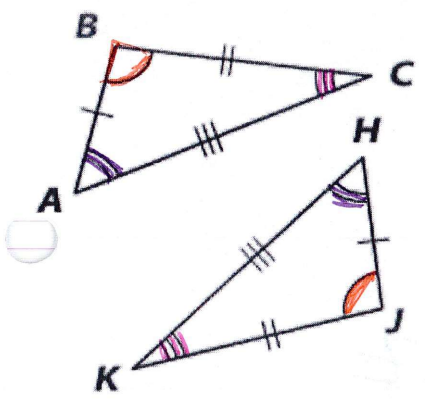
Congruence Statement

$$\triangle ABC \cong \triangle HJK$$

Model



Congruency Statements show which parts of shapes are congruent. Take $\triangle ABC$ and $\triangle HKJ$:



This is a **VALID** congruency statement because it links each congruent angle.

Valid Statement

$$\triangle BCA \cong \triangle JKH$$

Use Colors

This is an **INVALID** congruency statement because it indicates Congruency between angles that are not congruent:

Not a Valid Statement

$$\triangle ABC \cong \triangle HKJ$$

FF all corresponding parts are congruent, then the figures are congruent:

(IFF = "if and only if")

$$ABCD \cong WXYZ$$

$$\angle A \cong \angle W$$

$$\angle B \cong \angle X$$

$$\angle C \cong \angle Y$$

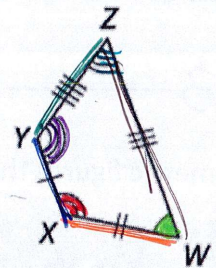
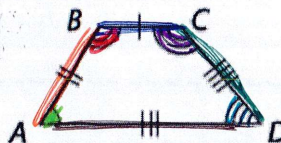
$$\angle D \cong \angle Z$$

$$\overline{AB} \cong \overline{XW}$$

$$\overline{AD} \cong \overline{ZW}$$

$$\overline{CD} \cong \overline{YZ}$$

$$\overline{BC} \cong \overline{XY}$$



In the diagram, $\triangle ABC \cong \triangle DFE$. Find the values of x and y .

$$\angle F \cong \angle B$$

CPCTC

$$m\angle F = m\angle B$$

Definition of congruence

$$8y - 5 = 99$$

Substitution *Blue*

$$8y = 104$$

Add 5 to each side.

$$y = 13$$

Divide each side by 8.

$$\overline{FE} \cong \overline{BC}$$

CPCTC

$$FE = BC$$

Definition of congruence

$$2y + x = 38.4$$

Substitution

$$2(13) + x = 38.4$$

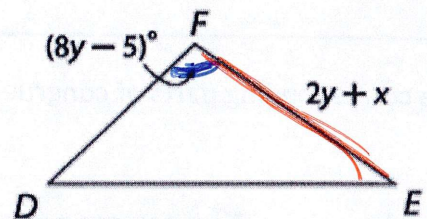
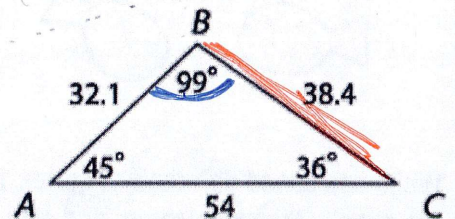
Substitution

$$26 + x = 38.4$$

Simplify.

$$x = 12.4$$

Subtract 26 from each side.

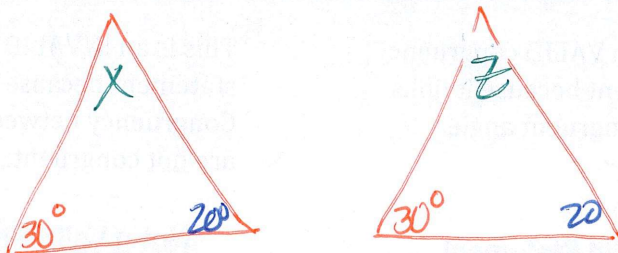


$$8y - 5 = 99$$

Third Angles Theorem:

If two angles of one triangle are congruent to two angles of a 2nd triangle, then the 3rd angles are also congruent.

Draw a picture and/or explain in your own words why this is true:



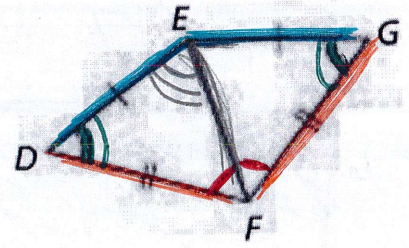
$$m\angle X = m\angle Z$$

Because

$$180 - 30 - 20 = m\angle X \quad 180 - 30 - 20 = m\angle Z$$

Write a two-column proof.

Given: $\overline{DE} \cong \overline{GE}$, $\overline{DF} \cong \overline{GF}$, $\angle D \cong \angle G$,
 $\angle DFE \cong \angle GFE$



Prove: $\triangle DEF \cong \triangle GEF$ *Prove*

- ① $\overline{DE} \cong \overline{GE}$ $\overline{DF} \cong \overline{GF}$ $\angle D \cong \angle G$
 $\angle DFE \cong \angle GFE$
- ② $\overline{EF} \cong \overline{EF}$
- ③ $\angle DEF \cong \angle GEF$
- ④ $\triangle DEF \cong \triangle GEF$

- ① Given
- ② Reflexive prop of Congruence
- ③ 3rd angles thm
- ④ Def of Cong Polygons
all corresponding parts cong.

As with equality, and congruence, the reflexive, Symmetric, and Transitive Properties hold true for triangles as well:

Reflexive - $AB = AB$
 Symmetric - If $A = B$ then $B = A$
 Transitive - if $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$ then $m\angle 1 = m\angle 3$
Remember these from ch 2

Theorem 4.4 Properties of Triangle Congruence

Reflexive Property of Triangle Congruence

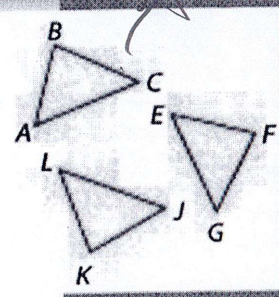
$\triangle ABC \cong \triangle ABC$

Symmetric Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

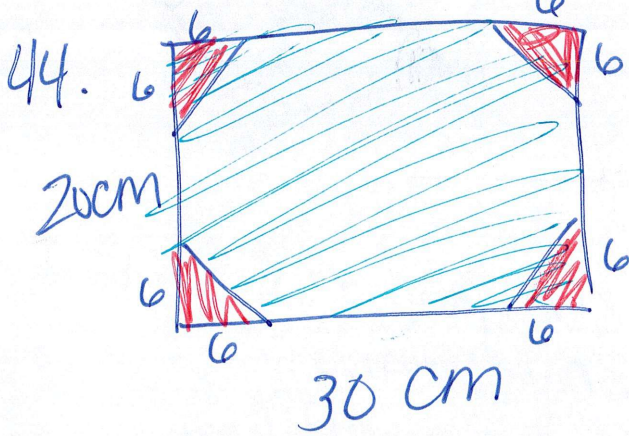
Transitive Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



Homework pg. 259 5-15 odd
 44-47

44-47



Area of octagon

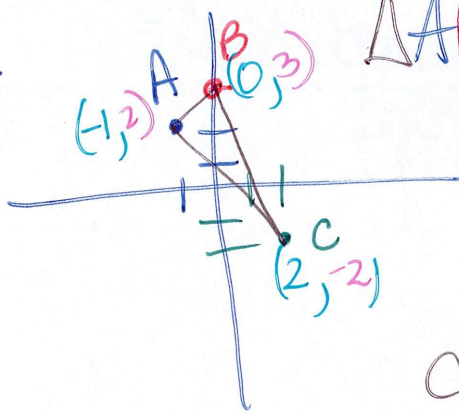
Find area of  Subtract out 4 of them

$$30 \times 20 - 4 \left(\frac{1}{2} \cdot 6 \cdot 6 \right)$$

$$600 - 72$$

528 cm² B

45.



$\triangle ABC \cong \triangle HIJ$

Measure of HJ will be the same as Measure of AC

CPCTC

$$\text{distance} = \sqrt{(-1-2)^2 + (2--2)^2}$$

$$D = \sqrt{9 + 16}$$

$$D = \sqrt{25}$$

Distance is 5

46. $x^2 + 19x - 42$ $\frac{-42}{2, -2 | 19}$
 $(x+21)(x-2)$ **H**

47. D

4.4

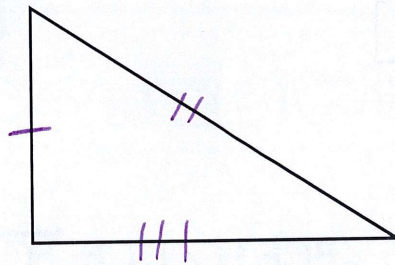
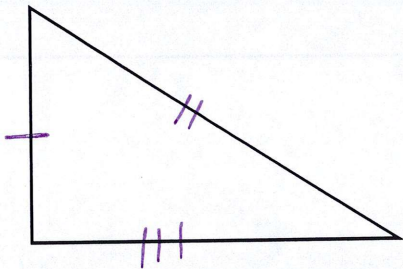
We know that we can prove that triangles are congruent if all their corresponding parts are congruent, but it's actually possible to prove the same thing with fewer known congruent parts

(as long as we have the correct congruent parts) *Really nice*

SIDE - SIDE - SIDE (SSS) Congruence: *always*

The first triangle congruency theorem we will look at is SSS (Side-Side-Side) congruence.

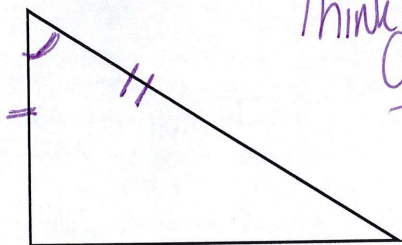
"If three sides of one triangle are congruent to the three sides of a 2nd triangle, the two triangles are congruent."



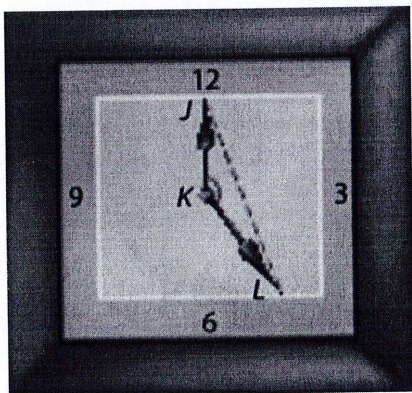
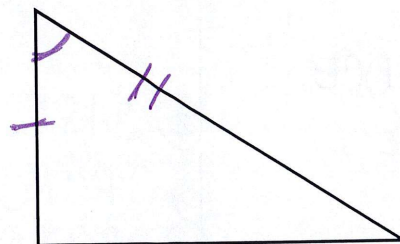
SIDE - ANGLE - SIDE (SAS) Congruence:

The second triangle congruency theorem we will use is ^{SAS}SSS (Side-Angle-Side) congruence.

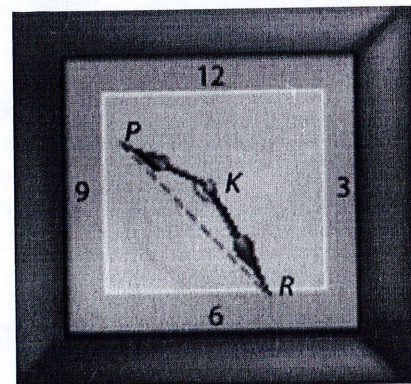
"If two sides of a triangle and their included angle are congruent with the corresponding sides/angle of a 2nd triangle, then the triangles are congruent."



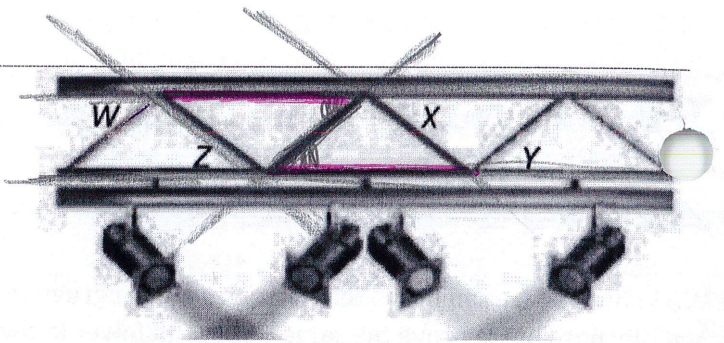
Think of a Clock



The triangle formed by the hands of this clock will always be congruent so long as angle K does not change



LIGHTING The scaffolding for stage lighting shown appears to be made up of congruent triangles. If $\overline{WX} \cong \overline{YZ}$ and $\overline{WX} \parallel \overline{ZY}$, write a two-column proof to prove that $\triangle WXZ \cong \triangle YZX$.



Proof:

Statements

1. $\overline{WX} \cong \overline{YZ}$

2. $\overline{WX} \parallel \overline{ZY}$

3. $\overline{ZX} = \overline{ZX}$

4. $\angle WXZ \cong \angle XZY$

5. $\triangle WXZ \cong \triangle YZX$

Reasons

1. Given

2. Given

3. Reflexive

4. alternate interior angles

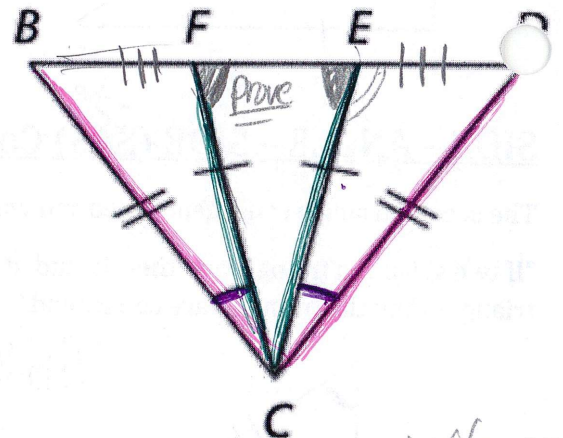
5. SAS

Think about this in parts

YOYO:

Given: $\overline{BC} \cong \overline{DC}$, $\angle BCF \cong \angle DCE$, $\overline{FC} \cong \overline{EC}$

Prove: $\angle CFD \cong \angle CEB$



Think of 2 different Δ 's

① $\overline{BC} \cong \overline{DC}$
 $\overline{FC} \cong \overline{EC}$
 $\angle BCF \cong \angle DCE$

① Given

② $\triangle BCF \cong \triangle DCE$

② SAS

③ $\overline{BF} \cong \overline{ED}$

③ CPCTC

④ $m\angle BFC = m\angle DEC$

④ CPCTC

⑤ $\angle BFC$; $\angle CFD$ are
 $\angle DEC$; $\angle CEB$ linear pairs

⑤ Def of linear pairs

⑥ $\angle BFC$; $\angle CFD$
 $\angle DEC$; $\angle CEB$ are supp

⑥ Def of linear pairs

⑦ $m\angle BFC + m\angle CFD = 180$
 $m\angle DEC + m\angle CEB = 180$

⑦ Def of Supp

⑧ $m\angle BFC + m\angle CFD = m\angle DEC + m\angle CEB$ ⑧ Substitution

⑨ $m\angle DEC + m\angle CFD = m\angle DEC + m\angle CEB$ ⑨ Subs.

⑩ $m\angle CFD = m\angle CEB$

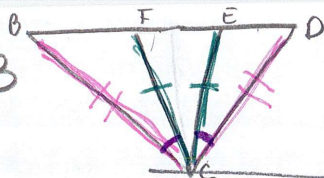
⑩ subtraction

⑪ $\angle CFD \cong \angle CEB$

⑪ Def \cong \angle 's

Flow chart
proof 4-4

Prove $\angle CFD \cong \angle CEB$



$$\overline{BC} \cong \overline{DC}$$

Given

$$\overline{FC} \cong \overline{EC}$$

Given

$$\angle BCF \cong \angle DCE$$

Given

$$\triangle BCF \cong \triangle DCE$$

SAS

$$\angle BFC \cong \angle DEC$$

CPCTC

$$m\angle BFC = m\angle DEC$$

Def \cong

$\angle DEC$ & $\angle CEB$
 $\angle BFC$ & $\angle CFD$
are linear pairs
def of linear pair

$\angle DEC$ & $\angle CEB$
 $\angle BFC$ & $\angle CFD$
are supplementary
def of linear pair

$$m\angle BFC + m\angle CFD = 180$$

$$m\angle DEC + m\angle CEB = 180$$

Opp of Supp

$$m\angle BFC + m\angle CFD = m\angle DEC + m\angle CEB$$

Substitution

$$m\angle DEC + m\angle CFD = m\angle DEC + m\angle CEB$$

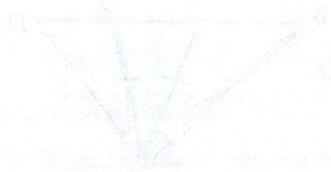
Substitution

$$m\angle CFD = m\angle CEB$$

Subtraction

$$\angle CFD \cong \angle CEB$$

Def of \cong



$\angle A = \angle B = \angle C = 60^\circ$
 $\angle A' = \angle B' = \angle C' = 120^\circ$

$\angle A + \angle A' = 180^\circ$
 $\angle B + \angle B' = 180^\circ$
 $\angle C + \angle C' = 180^\circ$

$\angle A' = 180^\circ - \angle A$
 $\angle B' = 180^\circ - \angle B$
 $\angle C' = 180^\circ - \angle C$

$\angle A' + \angle B' + \angle C' = 180^\circ - \angle A + 180^\circ - \angle B + 180^\circ - \angle C$
 $= 540^\circ - (\angle A + \angle B + \angle C)$

$\angle A' + \angle B' + \angle C' = 540^\circ - 180^\circ = 360^\circ$

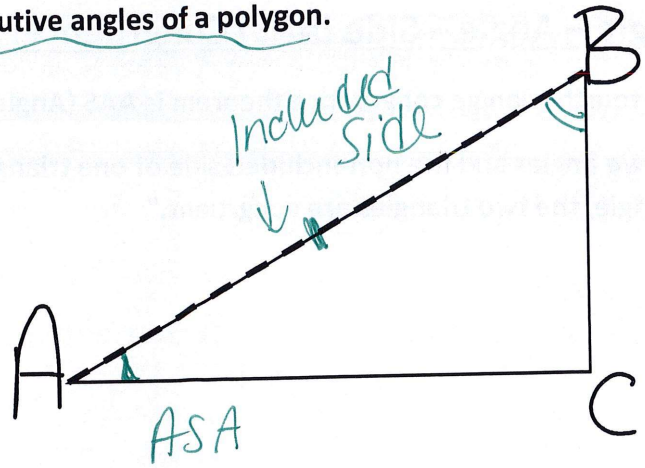
$\angle A' + \angle B' + \angle C' = 360^\circ$

$\angle A' + \angle B' + \angle C' = 360^\circ$

$\angle A' + \angle B' + \angle C' = 360^\circ$

An Included Side is the side located between two consecutive angles of a polygon.

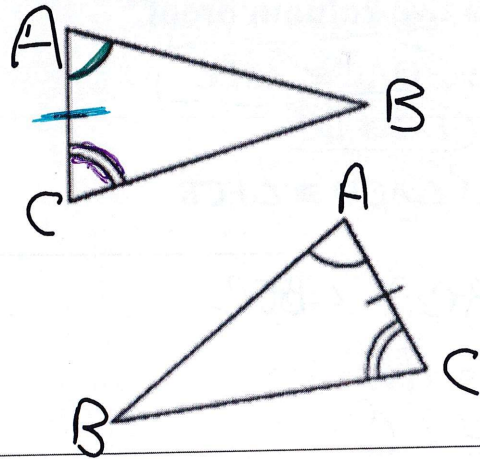
$\triangle ABC$, \overline{AB} is the included side between $\angle A$ and $\angle B$



ANGLE - SIDE - ANGLE (ASA) Congruence:

The third triangle congruency theorem we will use is ASA (Angle-Side-Angle) congruence.

"If two angles and their included side of one triangle are congruent to the corresponding parts of a 2nd triangle, the two triangles are congruent."

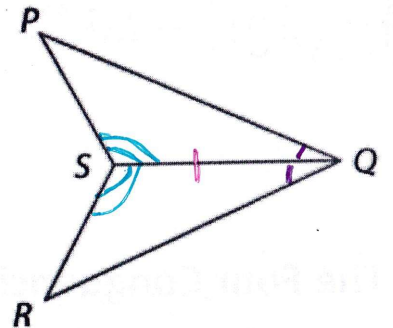


Write a two-column proof.

Given: \overline{QS} bisects $\angle PQR$;

$$\angle PSQ \cong \angle RSQ.$$

Prove: $\triangle PQS \cong \triangle RQS$



Proof:

① \overline{QS} bisects $\angle PQR$

② $\angle PQS \cong \angle RQS$

③ $\angle PSQ \cong \angle RSQ$

\Rightarrow ④ $\overline{SQ} \cong \overline{QS}$

⑤ $\triangle PQS \cong \triangle RQS$

① Given

② Def of angle bisector

③ Given

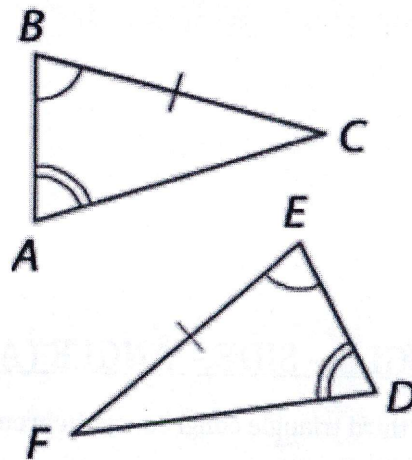
④ Reflexive prop

⑤ ASA

Angle – Angle – Side (AAS) Congruence:

The fourth triangle congruence theorem is AAS (Angle-Angle-Side) Congruence.

“If two angles and the non-included side of one triangle are congruent to the corresponding parts of a 2nd triangle, the two triangles are congruent.”

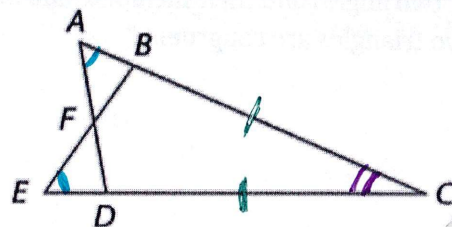


Write a two-column proof.

Given: $\angle DAC \cong \angle BEC$

$\overline{DC} \cong \overline{BC}$

Prove: $\triangle ACD \cong \triangle ECB$



- ① $\angle DAC \cong \angle BEC$
- ② $\overline{DC} \cong \overline{BC}$
- ③ $\angle C \cong \angle C$
- ④ $\triangle ACD \cong \triangle ECB$

- ① Given
- ② Given
- ③ Reflexive
- ④ AAS

The Four Congruencies:

| SSS | SAS | ASA | AAS |
|---|---|---|--|
| | | | |
| Three pairs of corresponding sides are congruent. | Two pairs of corresponding sides and their included angles are congruent. | Two pairs of corresponding angles and their included sides are congruent. | Two pairs of corresponding angles and the corresponding nonincluded sides are congruent. |

What's Missing?

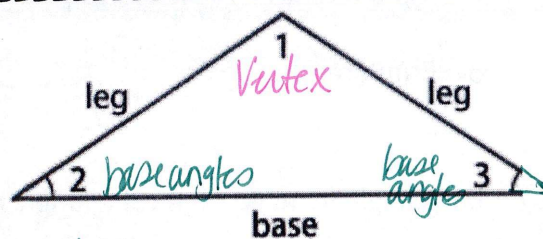
4.6

Isosceles Δ have at least two congruent sides.

The two congruent sides are called legs.

The angle joining the congruent sides is called the Vertex \angle .

The angles that join the 3rd side with the congruent legs are the base \angle 's.



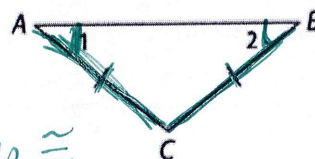
Theorems Isosceles Triangle

4.10 Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.

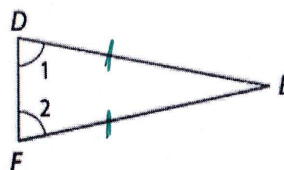
base angles \cong



4.11 Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\overline{FE} \cong \overline{DE}$.

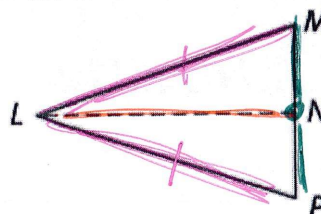


To prove the Isosceles Triangle Theorem, draw an auxiliary line and use the two triangles formed:

Proof Isosceles Triangle Theorem

Given: $\triangle LMP$; $\overline{LM} \cong \overline{LP}$

Prove: $\angle M \cong \angle P$



Proof:

Statements

1. Let N be the midpoint of \overline{MP} .
2. Draw an auxiliary segment \overline{LN} .
3. $\overline{LM} \cong \overline{LP}$
4. $\overline{MN} \cong \overline{PN}$
5. $\overline{LN} = \overline{NL}$
6. $\triangle MNL \cong \triangle PNL$
7. $\angle M \cong \angle P$

Reasons

1. Every segment has exactly one midpoint.
2. Two points determine a line.
3. Given
4. Def of Midpoint
5. Reflexive prop
6. SSS
7. CPCTC

Equilateral Triangles:

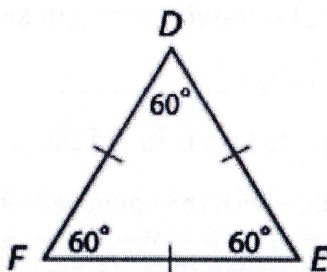
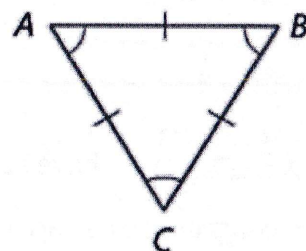
• A triangle is equilateral IFF it is equiangular.

○ Equiangular: $\angle A \cong \angle B \cong \angle C$

○ Equilateral: $\overline{AB} \cong \overline{BC} \cong \overline{AC}$

• Each angle in an equilateral triangle measures 60°

Must be



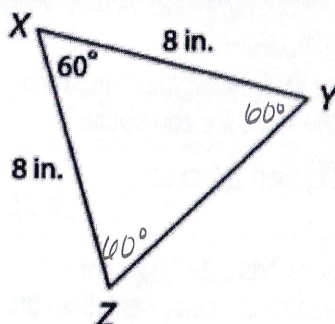
Solve the Triangle:

(find each missing measure)

$$\angle Y = 60^\circ$$

$$\angle Z = 60^\circ$$

$$\overline{XY} = 8 \text{ in}$$

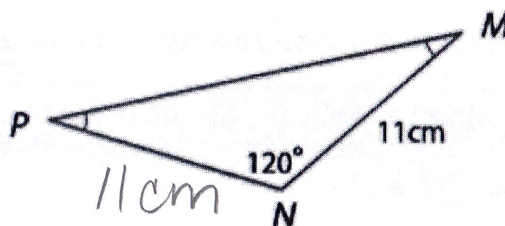


$$\angle M = 180 - 120 = 60 \div 2 = 30^\circ$$

$$\angle P = 30^\circ$$

$$\overline{PN} = 11 \text{ cm}$$

$\overline{PM} =$ Can't determine

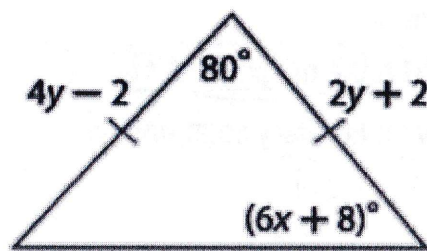


$$c = 180 - 80 = 100$$

$$6x + 8 + 6x + 8 = 100$$

$$\begin{array}{r} 12x + 16 = 100 \\ -16 \quad -16 \\ \hline 12x = 84 \end{array}$$

$$x = 7$$



$$\begin{array}{r} 4y - 2 = 2y + 2 \\ -2y \quad -2y \\ \hline 2y - 2 = 2 \end{array}$$

$$\begin{array}{r} 2y - 2 = 2 \\ 2y = 4 \end{array} \quad y = 2$$

Perform the following proofs:

1) Given: $\angle RXT \cong \angle XST$
 XT bisects $\angle RTS$
 Prove: $\triangle RXT \cong \triangle SXT$

① $\angle RXT \cong \angle XST$ ① Given
 ② XT bisects $\angle RTS$ ② Given
 ③ $\angle XTR \cong \angle XTS$ ③ Def angle Bisector
 ④ $\overline{XT} \cong \overline{TX}$ ④ Reflexive
 ⑤ $\triangle RXT \cong \triangle SXT$ ⑤ AAS

Oct 14-8:21 AM

2) Given: $AB \parallel CD$
 X is the mid-point of AD
 Prove: $BX \cong CX$

① X is Midpoint of AD ① Given
 ② $AX \cong DX$ ② Def of Midpoint
 ③ $AB \parallel CD$ ③ Given
 ④ $\angle ABX \cong \angle DCX$ ④ Alternate Interior \angle 's
 ⑤ $\angle AXB \cong \angle DXC$ ⑤ Vertical Angles
 ⑥ $\triangle AXB \cong \triangle DXC$ ⑥ AAS
 ⑦ $BX \cong CX$ ⑦ CPCTC

Oct 14-9:09 AM

3) Given: $XY \parallel WZ$
 $XW \parallel YZ$
 Prove: $\angle XWZ \cong \angle ZYX$

① $XY \parallel WZ$ ① Given
 ② $\angle YXZ \cong \angle WZX$ ② Alternate Interior
 ③ $XW \parallel YZ$ ③ Given
 ④ $\angle WXZ \cong \angle YZX$ ④ Alternate Interior
 ⑤ $\overline{XZ} = \overline{ZX}$ ⑤ Reflexive
 ⑥ $\triangle XWZ \cong \triangle ZYX$ ⑥ ASA
 ⑦ $\angle XWZ \cong \angle ZYX$ ⑦ CPCTC

Oct 14-9:09 AM

4) Given: $\angle WXO \cong \angle ZYO$
 $XO \cong YO$
 Prove: $\overline{WO} \cong \overline{ZO}$

① $\angle WXO \cong \angle ZYO$ ① Given
 ② $XO \cong YO$ ② Given
 ③ $\angle WOX \cong \angle ZOY$ ③ Vertical Angles
 ④ $\triangle WOX \cong \triangle ZOY$ ④ ASA
 ⑤ $\overline{WO} \cong \overline{ZO}$ ⑤ CPCTC

Oct 14-9:09 AM

5) Given: $DF \parallel EG$
 G is the midpoint of FH
 $\angle DGF \cong \angle EHG$
 Prove: $DG \cong EH$

① $DF \parallel EG$ ① Given
 ② $\angle DFG \cong \angle EGH$ ② Corresponding \angle 's
 ③ G is the Midpoint of FH ③ Given
 ④ $\overline{FG} \cong \overline{GH}$ ④ Def of Midpoint
 ⑤ $\angle DGF \cong \angle EHG$ ⑤ Given
 ⑥ $\triangle DGF \cong \triangle EHG$ ⑥ ASA
 ⑦ $DG \cong EH$ ⑦ CPCTC

Oct 14-9:09 AM

① Given
 ② Corresponding \angle 's
 ③ Given
 ④ Def of Midpoint
 ⑤ Given
 ⑥ ASA
 ⑦ CPCTC

1. Water Right
 2. Priority
 3. Quantity
 4. Point of Diversion
 5. Use
 6. Beneficial Use
 7. Appropriation
 8. Priority
 9. Quantity
 10. Point of Diversion
 11. Use
 12. Beneficial Use
 13. Appropriation
 14. Priority
 15. Quantity
 16. Point of Diversion
 17. Use
 18. Beneficial Use
 19. Appropriation
 20. Priority
 21. Quantity
 22. Point of Diversion
 23. Use
 24. Beneficial Use
 25. Appropriation

26. Priority
 27. Quantity
 28. Point of Diversion
 29. Use
 30. Beneficial Use
 31. Appropriation
 32. Priority
 33. Quantity
 34. Point of Diversion
 35. Use
 36. Beneficial Use
 37. Appropriation
 38. Priority
 39. Quantity
 40. Point of Diversion
 41. Use
 42. Beneficial Use
 43. Appropriation
 44. Priority
 45. Quantity
 46. Point of Diversion
 47. Use
 48. Beneficial Use
 49. Appropriation
 50. Priority

51. Priority
 52. Quantity
 53. Point of Diversion
 54. Use
 55. Beneficial Use
 56. Appropriation
 57. Priority
 58. Quantity
 59. Point of Diversion
 60. Use
 61. Beneficial Use
 62. Appropriation
 63. Priority
 64. Quantity
 65. Point of Diversion
 66. Use
 67. Beneficial Use
 68. Appropriation
 69. Priority
 70. Quantity
 71. Point of Diversion
 72. Use
 73. Beneficial Use
 74. Appropriation
 75. Priority
 76. Quantity
 77. Point of Diversion
 78. Use
 79. Beneficial Use
 80. Appropriation

81. Priority
 82. Quantity
 83. Point of Diversion
 84. Use
 85. Beneficial Use
 86. Appropriation
 87. Priority
 88. Quantity
 89. Point of Diversion
 90. Use
 91. Beneficial Use
 92. Appropriation
 93. Priority
 94. Quantity
 95. Point of Diversion
 96. Use
 97. Beneficial Use
 98. Appropriation
 99. Priority
 100. Quantity
 101. Point of Diversion
 102. Use
 103. Beneficial Use
 104. Appropriation
 105. Priority
 106. Quantity
 107. Point of Diversion
 108. Use
 109. Beneficial Use
 110. Appropriation

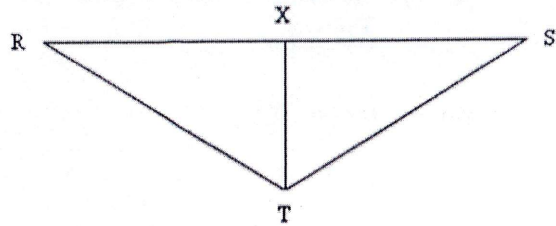
111. Priority
 112. Quantity
 113. Point of Diversion
 114. Use
 115. Beneficial Use
 116. Appropriation
 117. Priority
 118. Quantity
 119. Point of Diversion
 120. Use
 121. Beneficial Use
 122. Appropriation
 123. Priority
 124. Quantity
 125. Point of Diversion
 126. Use
 127. Beneficial Use
 128. Appropriation
 129. Priority
 130. Quantity
 131. Point of Diversion
 132. Use
 133. Beneficial Use
 134. Appropriation
 135. Priority
 136. Quantity
 137. Point of Diversion
 138. Use
 139. Beneficial Use
 140. Appropriation

141. Priority
 142. Quantity
 143. Point of Diversion
 144. Use
 145. Beneficial Use
 146. Appropriation
 147. Priority
 148. Quantity
 149. Point of Diversion
 150. Use
 151. Beneficial Use
 152. Appropriation
 153. Priority
 154. Quantity
 155. Point of Diversion
 156. Use
 157. Beneficial Use
 158. Appropriation
 159. Priority
 160. Quantity
 161. Point of Diversion
 162. Use
 163. Beneficial Use
 164. Appropriation
 165. Priority
 166. Quantity
 167. Point of Diversion
 168. Use
 169. Beneficial Use
 170. Appropriation

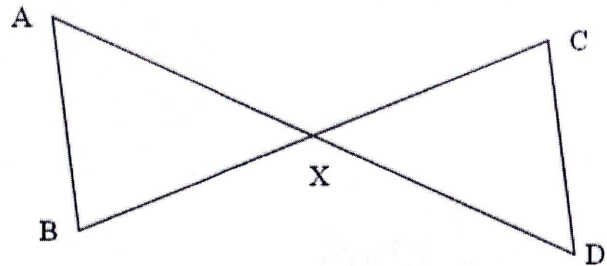
Name: _____

Perform the following proofs:

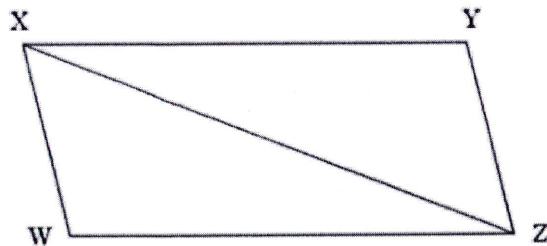
- 1) Given: $\angle XRT \cong \angle XST$
 \overline{XT} bisects $\angle RTS$
- Prove: $\triangle RXT \cong \triangle SXT$



- Given: $AB \parallel CD$
X is the mid-point of AD
- Prove: $BX \cong CX$

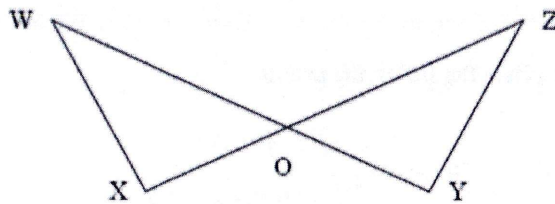


- 3) Given: $XY \parallel WZ$,
 $XW \parallel YZ$
- Prove: $\angle XWZ \cong \angle ZYX$



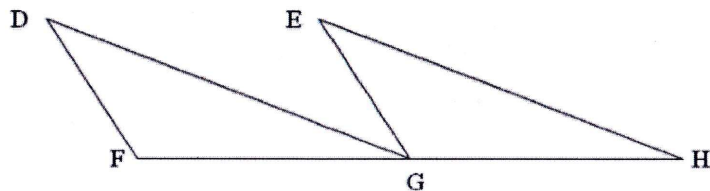
4) Given: $\angle W XO \cong \angle Z YO$
 $\overline{XO} \cong \overline{YO}$

Prove: $\overline{WO} \cong \overline{ZO}$



5) Given: $\overline{DF} \parallel \overline{EG}$
G is the midpoint of FH
 $\angle DGF \cong \angle EHG$

Prove: $\overline{DG} \cong \overline{EH}$



4.4 Proofs SSS SAS

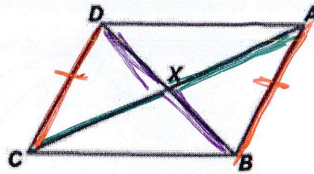
Write a flow proof.

Given: X is the midpoint of \overline{BD} .

X is the midpoint of \overline{AC} .

$\overline{CD} \cong \overline{BA}$

Prove: $\triangle DXC \cong \triangle BXA$



Flow proof for $\triangle DXC \cong \triangle BXA$:

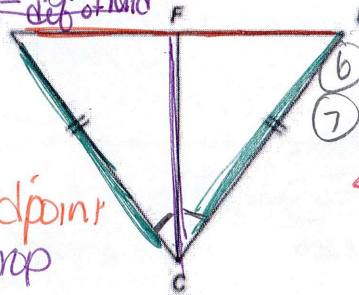
- Given: $\overline{CD} \cong \overline{BA}$
- Given: X is Midpoint of \overline{AC} → $\overline{CX} \cong \overline{AX}$ (def of Midpoint)
- Given: X is Midpoint of \overline{BD} → $\overline{DX} \cong \overline{BX}$ (def of Midpoint)
- Conclusion: $\triangle DXC \cong \triangle BXA$ (SSS)

Given: $\overline{BC} \cong \overline{DC}$, F is the midpoint of \overline{BD}

Prove: \overline{FC} bisects $\angle BCD$

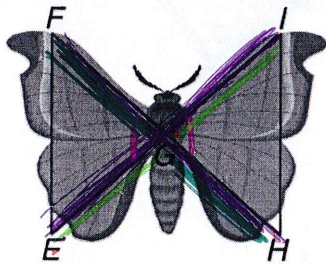
- $\overline{BC} \cong \overline{DC}$
- F is the midpoint of \overline{BD}
- $\overline{BF} \cong \overline{DF}$
- $\overline{FC} \cong \overline{CF}$
- $\triangle BFC \cong \triangle DFC$

- Given
- Given
- Def of Midpoint
- Reflexive prop
- SSS



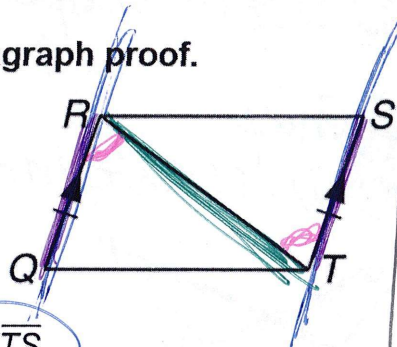
- $\angle FCB \cong \angle FCD$ (CPCTC)
- \overline{FC} bisects $\angle BCD$ (Def of angle bisector)

ENTOMOLOGY The wings of one type of moth form two triangles. Write a two-column proof to prove that $\triangle FEG \cong \triangle HIG$ if $\overline{EI} \cong \overline{FH}$, and G is the midpoint of both \overline{EI} and \overline{FH} .



| Statements | Reasons |
|--|-------------------|
| ① $\overline{EI} \cong \overline{FH}$ | ① Given |
| ② $\angle FGE \cong \angle HGI$ | ② Vertical angles |
| ③ G is Midpoint of \overline{EI} and \overline{FH} | ③ Given |
| ④ $\overline{EG} \cong \overline{IG}$ | ④ Def of Midpoint |
| ⑤ $\overline{FG} \cong \overline{HG}$ | ⑤ Def of Midpoint |
| ⑥ $\triangle FEG \cong \triangle HIG$ | ⑥ SAS |

Write a paragraph proof.



Given: $\overline{RQ} \parallel \overline{TS}$

$\overline{RQ} \cong \overline{TS}$

Prove: $\angle Q \cong \angle S$

We know that $\overline{RQ} \cong \overline{TS}$ and that $\overline{RQ} \parallel \overline{TS}$ because it is given. Because of the reflexive property we know that $\overline{RT} \cong \overline{TR}$. Angle $\angle QRT$ and $\angle STR$ are alternate interior angles so they are congruent. We can say $\triangle QRT \cong \triangle STR$ because of SAS. Congruent parts of congruent triangles are \cong . So $\angle Q \cong \angle S$.

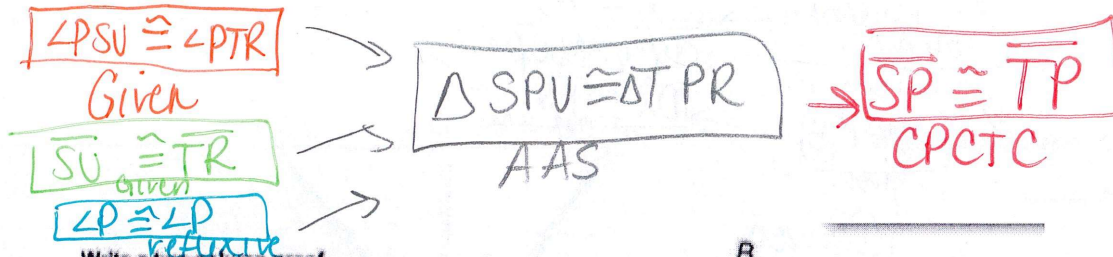
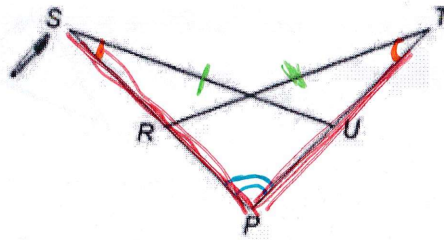
4.5 Proofs AAS ASA

Write a paragraph proof.

Given: $\angle PSU \cong \angle PTR$

$SU \cong TR$

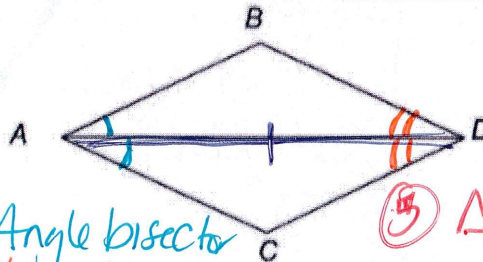
Prove: $SP \cong TP$



Write a two-column proof.

Given: \overline{AD} bisects $\angle BAC$ and $\angle BDC$.

Prove: $\Delta ABD \cong \Delta ACD$

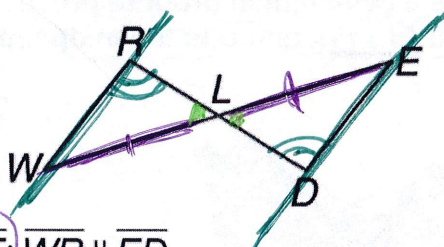


- | | | |
|---|--|--------------------------------------|
| <ol style="list-style-type: none"> ① \overline{AD} bisects $\angle BAC$; $\angle BDC$ ② $\angle BAD \cong \angle CAD$ ③ $\angle BDA \cong \angle CDA$ ④ $\overline{AD} \cong \overline{DA}$ | <ol style="list-style-type: none"> ① Given ② Def of Angle bisector ③ Def of Angle bisector ④ Reflexive | $\Delta ABD \cong \Delta ACD$ ASA |
|---|--|--------------------------------------|

Write a two-column proof.

Given: L is the midpoint of \overline{WE} ; $\overline{WR} \parallel \overline{ED}$.

Prove: $\Delta WRL \cong \Delta EDL$



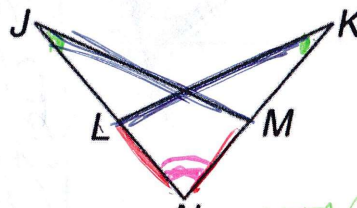
- | | | |
|--|--|--------------------------------------|
| <ol style="list-style-type: none"> ① L is Midpoint of \overline{WE} ② $\overline{WL} \cong \overline{EL}$ ③ $\angle RLW \cong \angle DLE$ ④ $\overline{WR} \parallel \overline{ED}$ ⑤ $\angle WRL \cong \angle EDL$ | <ol style="list-style-type: none"> ① Given ② Def of Midpoint ③ vertical angles ④ Given ⑤ alternate interior \angle's | $\Delta WRL \cong \Delta EDL$ AAS |
|--|--|--------------------------------------|

Write a paragraph proof.

Given: $\angle NKL \cong \angle NJM$

$\overline{KL} \cong \overline{JM}$

Prove: $\overline{LN} \cong \overline{MN}$



We know that $\angle NKL \cong \angle NJM$ and $\overline{KL} \cong \overline{JM}$ because it was given we also know that $\angle LNK \cong \angle JNM$ because of the reflexive property. So we can say $\Delta KNL \cong \Delta JNM$ because of AAS. We know that $\overline{LN} \cong \overline{MN}$ because CPCTC.