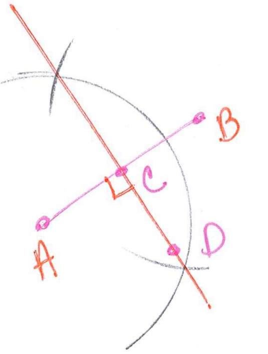
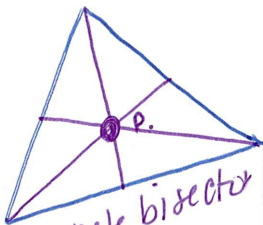
HONORS MATH 2 CHAPTER LESSON 5.1



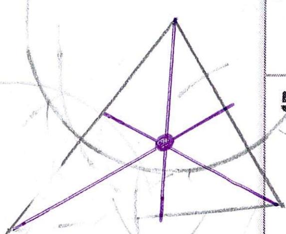
New Vocabulary

Construct w/ compass prot

- perpendicular bisector Make $\overline{AC} \cong \overline{BC}$ and $\angle ACD$ Right
- concurrent lines: 3 or more lines intersect at a common pt. they are called concurrent lines
- point of concurrency The point where concurrent lines intersect
- circumcenter The point of concurrency of the perpendicular bisectors is called the circumcenter of Δ.
- incenter The angle bisectors of a Δ are concurrent, and their pt of concurrency is called the incenter



↑ angle bisector



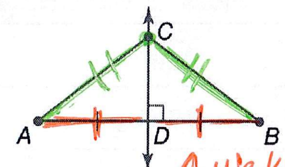
Incenter

Theorems Perpendicular Bisectors

5.1 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Example: If $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$, then $AC = BC$.

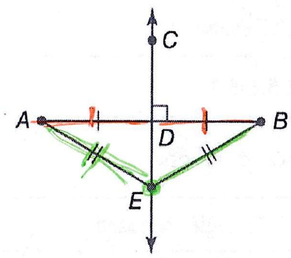


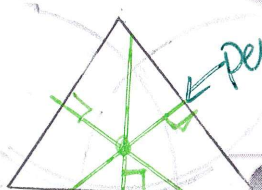
↑ We know that cause of per bisect

5.2 Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

Example: If $AE = BE$, then E lies on $\overleftrightarrow{CD}$, the $\perp$ bisector of $\overline{AB}$.





← Perpendicular bisector

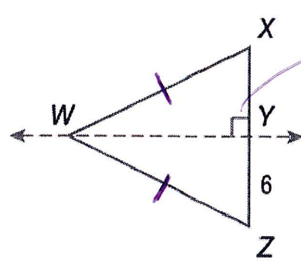
Circumcenter

EXAMPLE 1 Use the Perpendicular Bisector Theorems

B. Find *XY*.

We know $\overline{WX} \cong \overline{WZ}$

$\overline{WY} \perp \overline{XZ}$ from picture



know perpendicular

From the Converse of Perp Bisc thm

we know $\overline{XY} \cong \overline{ZY}$ because it is a Perp bisc

So $XY = 6$

Honors 2

Chapter 5 - Relationships in Triangles

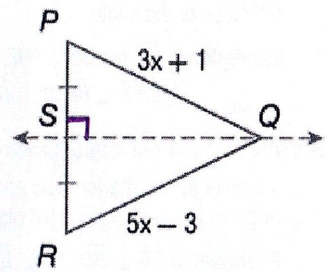
Day	Topic	Assignment	Present Problem
1	5.1 Bisectors of Triangles	pg.329 5,8,9,14,23,25, 27-30 55-58	
2	5.2 Medians and Altitudes of Tri's	pg.340 15-31 odd 44-47	
3	5.1-5.2 Review	WKSH	
4	**QUIZ 5.1-5.2**		
5	5.4 Indirect Proofs		
6	5.3/5.5 Triangle Inequalities		
7	5.6 Inequalities in Two Triangles and Inequalities Review		
8	**QUIZ 5.3 - 5.6**		
9	Review		
10	**TEST Chapter 5**		

HONORS MATH 2 CHAPTER LESSON 5.1

EXAMPLE 1 Use the Perpendicular Bisector Theorems

C. Find *PQ*.

$\overleftrightarrow{SQ}$ is the perpendicular bisector of $\overline{PR}$.



I know $PQ = RQ$

$3x+1 = 5x-3$

$-3x \quad -3x$

$1 = 2x - 3$

$+3 \quad +3$

$4 = 2x$

$x = 2$

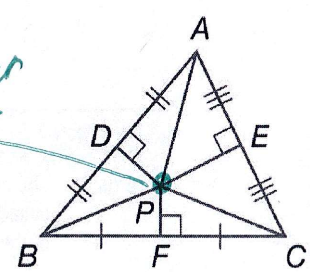
$PQ = 3(2)+1$

$PQ = 7$

Theorem 5.3 Circumcenter Theorem

Words The perpendicular bisectors of a triangle intersect at a point called the *circumcenter* that is equidistant from the vertices of the triangle.

Example If *P* is the circumcenter of $\triangle ABC$, then $PB = PA = PC$.



Circumcenter

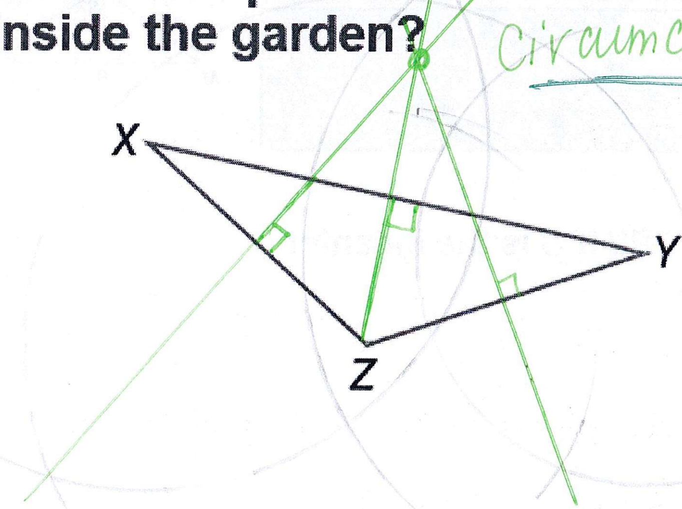
Real-World Example 2 Use the Circumcenter Theorem

Construct Circumcenter

Perp. bisectors

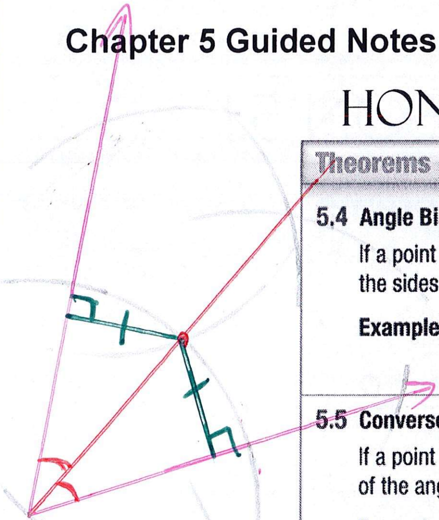
GARDEN A triangular-shaped garden is shown. Can a fountain be placed at the circumcenter and still be inside the garden?

Circumcenter



Because the Circumcenter of an obtuse Δ is outside the Δ it can't be Inside Garden

HONORS MATH 2 CHAPTER LESSON 5.1

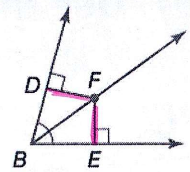


Theorems Angle Bisectors

5.4 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

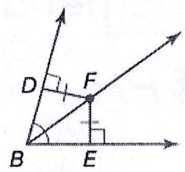
Example: If $\overrightarrow{BF}$ bisects $\angle DBE$, $\overline{FD} \perp \overrightarrow{BD}$, and $\overline{FE} \perp \overrightarrow{BE}$, then $DF = FE$.



5.5 Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.

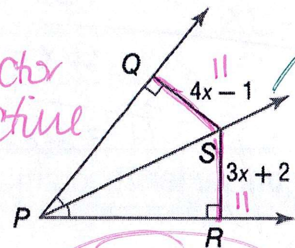
Example: If $\overline{FD} \perp \overrightarrow{BD}$, $\overline{FE} \perp \overrightarrow{BE}$, and $DF = FE$, then $\overrightarrow{BF}$ bisects $\angle DBE$.



EXAMPLE 3 Use the Angle Bisector Theorems

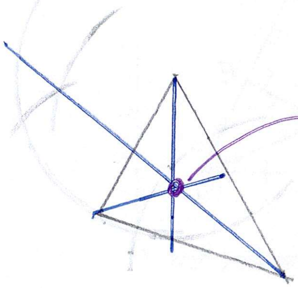
C. Find *QS*.

I know PS is an angle bisector of ∠QPR because of picture
∴ QS = RS because S is on the ∠ bisector



$4x-1 = 3x+2$
$-3x \quad -3x$
$x-1 = 2$
$+1 \quad +1$
$x = 3$

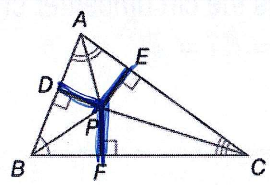
QS = 11
RS = 11



Theorem 5.6 Incenter Theorem

Words The angle bisectors of a triangle intersect at a point called the *incenter* that is equidistant from the sides of the triangle.

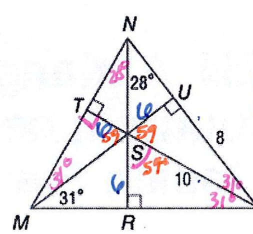
Example If *P* is the incenter of $\triangle ABC$, then $PD = PE = PF$.



EXAMPLE 4 Use the Incenter Theorem

A. Find *ST* if *S* is the incenter of Δ*MNP*.

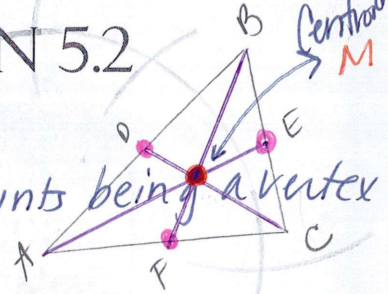
ST = 6
SU = 6
SR = 6



$8^2 + a^2 = 10^2$
$64 + a^2 = 100$
$-64 \quad -64$
$\sqrt{a^2} = \sqrt{36}$
$a = \pm 6$

B. Find *m∠SPU* if *S* is the incenter of Δ*MNP*.

HONORS MATH 2 CHAPTER LESSON 5.2



New Vocabulary

- **median** of a triangle is a segment with endpoints being a vertex of a Δ and Midpoints of opposite Sides

DC AE FB all Medians

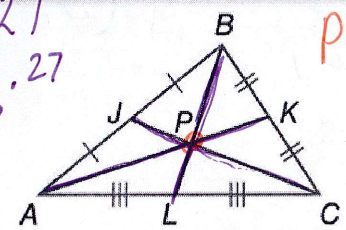
- **centroid** Point where all Medians Intersect
- **altitude** of Δ is a segment from a vertex to the line containing the opposite side and perpend. to the line containing that side can be Interior or Exterior of a Δ
- **orthocenter** point where the altitudes of a Δ Intersect

Theorem 5.7 Centroid Theorem

The medians of a triangle intersect at a point called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.

Example If P is the centroid of $\triangle ABC$, then $AP = \frac{2}{3}AK$, $BP = \frac{2}{3}BL$, and $CP = \frac{2}{3}CJ$.

$AP = \frac{2}{3}AK$



$AK = 27$
$AP = \frac{2}{3} \cdot 27$
$AP = 18$

P is Centroid

$BL = 18$
$BP = \frac{2}{3} \cdot 18$
$BP = 12$

$JC = 21$
$PC = \frac{2}{3} \cdot 21$
$PC = 14$

EXAMPLE 1 Use the Centroid Theorem

In ΔXYZ, P is the centroid and $YV = 12$. Find YP and PV.

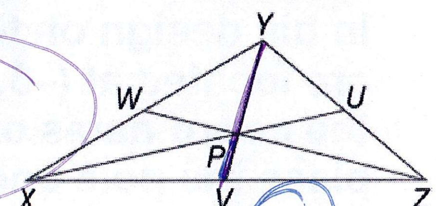
$YZ = 12$

So

$YP = \frac{2}{3}$ of 12

$\frac{2}{3} \cdot 12 = 8$

$YP = 8$



$PV = 12 - 8 = 4$

EXAMPLE 2 Use the Centroid Theorem

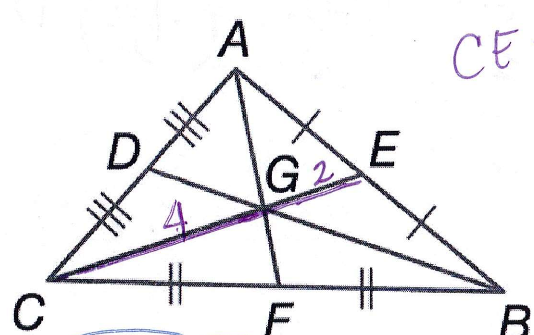
In ΔABC, $CG = 4$. Find GE.

$CE = 6$

$CG = \frac{2}{3} \cdot CE$

$\frac{3}{2} \cdot 4 = \frac{2}{3} \cdot CE \cdot \frac{3}{2}$

$\boxed{6 = CE}$



$GE = 2$

$24 = x$

HONORS MATH 2 CHAPTER LESSON 5.2

Real-World Example 3 **Find the Centroid on a Coordinate Plane**

SCULPTURE An artist is designing a sculpture that balances a triangle on top of a pole. In the artist's design on the coordinate plane, the vertices are located at (1, 4), (3, 0), and (3, 8). What are the coordinates of the point where the artist should place the pole under the triangle so that it will balance?

A B C

Balances

Distance $\overline{BC}$

$\left(\frac{3+3}{2}, \frac{0+8}{2}\right)$ $(3, 4)$

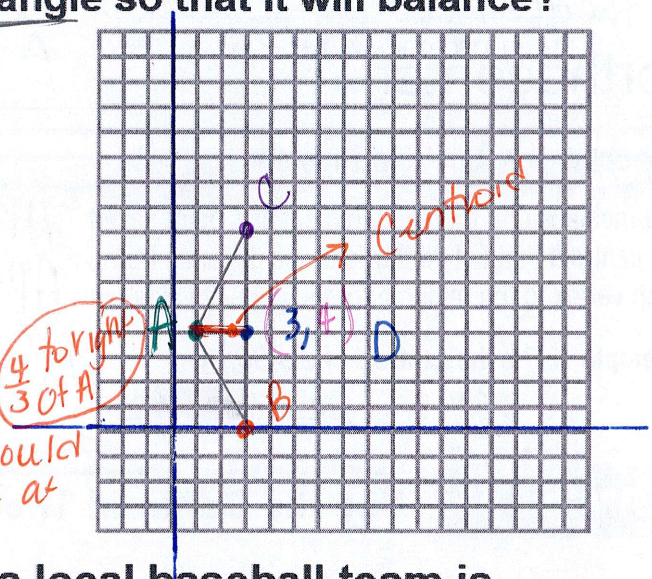
Distance of orange AD = 2

Centroid is $\frac{2}{3}(AD) = \frac{2}{3} \cdot 2 = \frac{4}{3}$ to right of A

add $1 + \frac{4}{3}$ $\left(1\frac{4}{3}, 4\right)$

$\frac{3}{3} + \frac{4}{3}$

The artist should place the pole at $\left(\frac{7}{3}, 4\right)$



BASEBALL A fan of a local baseball team is designing a triangular sign for the upcoming game. In his design on the coordinate plane, the vertices are located at (–3, 2), (–1, –2), and (–1, 6). What are the coordinates of the point where the fan should place the pole under the triangle so that it will balance?

A B C

Find Midpoint CB =

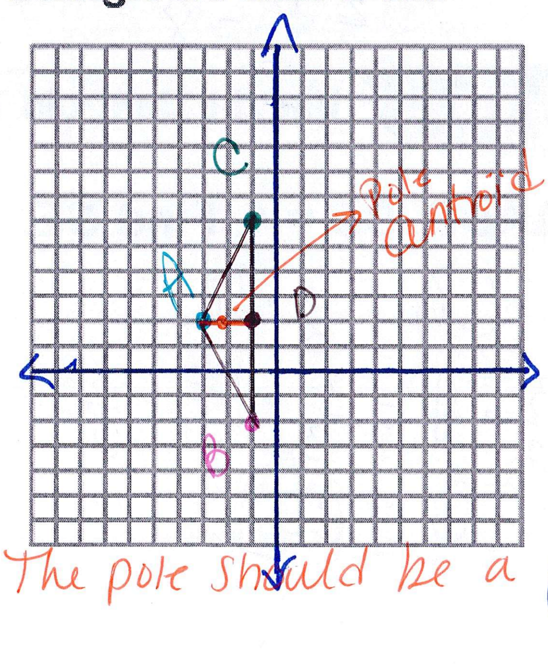
$\left(\frac{-1+-1}{2}, \frac{-2+6}{2}\right)$ $(-1, 2)$ D

Find distance of AD = 2

Centroid is $\frac{2}{3} \cdot 2 = \frac{4}{3}$ to right of A,

$-3 + \frac{4}{3}$ $\left(-1\frac{2}{3}, 2\right)$

$-\frac{9}{3} + \frac{4}{3} = \frac{-5}{3}$ $\rightarrow \left(-\frac{5}{3}, 2\right)$



The pole should be a $\left(\frac{-5}{3}, 2\right)$

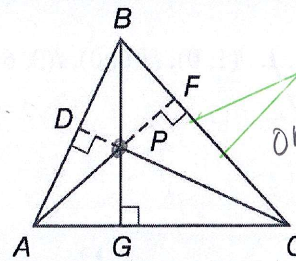
Altitude

HONORS MATH 2 CHAPTER LESSON 5.2

KeyConcept Orthocenter

The lines containing the altitudes of a triangle are concurrent, intersecting at a point called the **orthocenter**.

Example The lines containing altitudes $\overline{AF}$, $\overline{CD}$, and $\overline{BG}$ intersect at P, the orthocenter of $\triangle ABC$.



orthocenter

COORDINATE GEOMETRY The vertices of ΔABC are $A(-2, 2)$, $B(4, 4)$, and $C(1, -2)$. Find the coordinates of the orthocenter of ΔABC.

find Slope AB

$m = \frac{2}{6} = \frac{1}{3} \perp -3$

$y = -3x + 1$

Slope BC

$m = \frac{6}{3}$ $m = 2 \perp m = -\frac{1}{2}$

Slope AC

$m = -\frac{4}{3} \perp m = \frac{3}{4}$

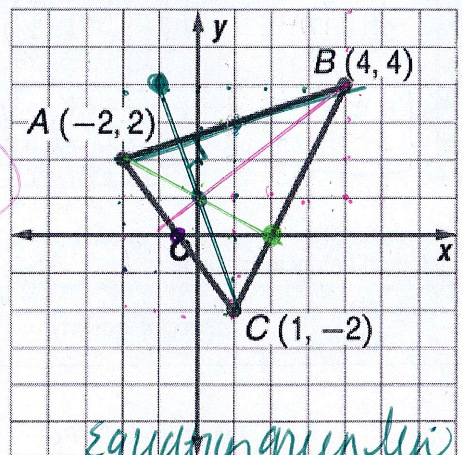
Orthocenter

A. (1, 0)

B. (0, 1)

C. (–1, 1)

D. (0, 0)



equation Pink

$y = \frac{3}{4}x + b$

$(4, 4)$

$4 = \frac{3}{4}(4) + b$

$4 = 3 + b$

$1 = b$

$y = \frac{3}{4}x + 1$

Equation green line

$y = -3x + b$

$-2 = -3(1) + b$

$-2 = -3 + b$

$+3 \quad +3$

$1 = b$

$y = -3x + 1$

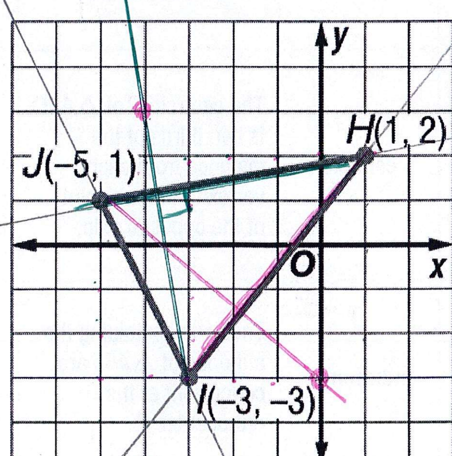
Intersect at y-intercept

COORDINATE GEOMETRY The vertices of ΔHIJ are $H(1, 2)$, $I(-3, -3)$, and $J(-5, 1)$. Find the coordinates of the orthocenter of ΔHIJ.

Slope JH $m = \frac{1}{6} \perp m = -\frac{6}{1}$

IH

$m = \frac{5}{4} \perp m = -\frac{4}{5}$



$y = -6x - 21$

$y = -\frac{4}{5}x - 3$

$-6x - 21 = -\frac{4}{5}x - 3$

$+\frac{4}{5}x \quad +\frac{4}{5}x$

$-5.2x - 21 = -3$

$-5.2x = 18$

$(-3.46, -.23)$ $\left(-3\frac{6}{13}, -\frac{3}{13}\right)$

$y = -6x + b$

$y = -6x - 21$

$(-4, 3)$

$3 = -6(-4) + b$

$3 = 24 + b$

$-24 \quad -24$

$-21 = b$

$y = -\frac{4}{5}x + b$

$(0, -3)$

$-3 = -\frac{4}{5}(0) +$

$y = -\frac{4}{5}x - 3$

HONORS MATH 2 CHAPTER LESSON 5.2

Exercises: EXTRA EXAMPLES IF NEEDED

COORDINATE GEOMETRY Find the coordinates of the orthocenter of the triangle with the given vertices.

altitudes perpendicular

1. $J(1, 0)$, $H(6, 0)$, $I(3, 6)$

$m = 3 \perp m = -\frac{1}{3}$

$y = -\frac{1}{3}x + 2$

$m = 3 \perp$

$m = +\frac{1}{3}$

$y = +\frac{1}{3}x + b$

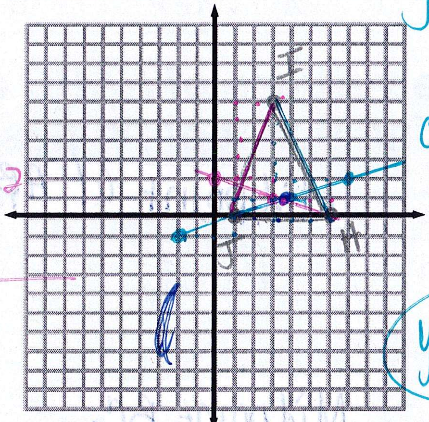
$(1, 0)$

$0 = +\frac{1}{3}(1) + b$

$0 = -\frac{1}{3} + b$

$-\frac{1}{3} = b$

$y = \frac{1}{3}x - \frac{1}{3}$



$\frac{1}{3}x - \frac{1}{3} = -\frac{1}{3}x + 2$

$+\frac{1}{3}x \quad +\frac{1}{3}x$

$\frac{2}{3}x - \frac{1}{3} = 2$

$+\frac{1}{3} \quad +\frac{1}{3}$

$\frac{2}{3}x = 2\frac{1}{3}$

$\frac{3}{2} \cdot \frac{2}{3}x = \frac{7}{3} \cdot \frac{3}{2}$

$x = \frac{7}{2}$

$(3\frac{1}{2}, \frac{5}{6})$

$y = -\frac{1}{3}(\frac{7}{2}) + 2$

$y = -\frac{7}{6} + \frac{2}{1}$

$y = -\frac{7}{6} + \frac{12}{6}$

$y = \frac{5}{6}$

2. $S(1, 0)$, $T(4, 7)$, $U(8, -3)$

ST slope $m = \frac{7}{3} \perp m = -\frac{3}{7}$

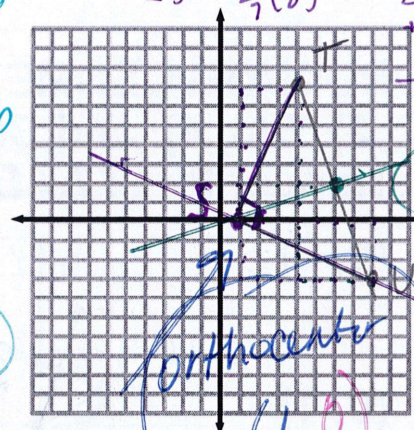
$y = -\frac{3}{7}x + b$

$-3 = -\frac{3}{7}(8) + b \qquad -3 = -\frac{24}{7} + b$

$-\frac{21}{7} + \frac{24}{7} \qquad +\frac{24}{7} \quad +\frac{24}{7}$

$\frac{3}{7} = b$

$y = -\frac{3}{7}x + \frac{3}{7}$



orthocenter $(1, 0)$

$m = -\frac{10}{4} \quad m = -\frac{5}{2}$

$\perp m = \frac{2}{5}$

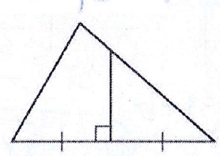
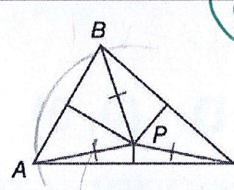
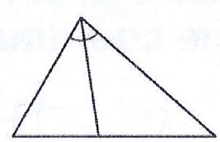
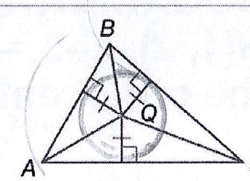
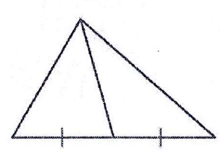
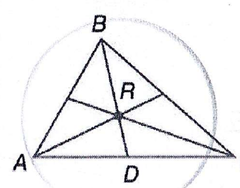
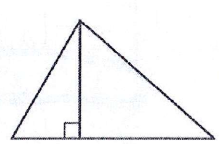
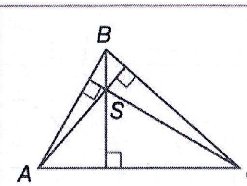
$y = \frac{2}{5}x + b$

$0 = \frac{2}{5}(1) + b$

$-\frac{2}{5} = b$

$y = \frac{2}{5}x - \frac{2}{5}$

ConceptSummary Special Segments and Points in Triangles

Name	Example	Point of Concurrency	Special Property	Example
Inside perpendicular bisector		Inside circumcenter	The circumcenter P of $\triangle ABC$ is equidistant from each vertex.	
Inside angle bisector		Inside incenter	The incenter Q of $\triangle ABC$ is equidistant from each side of the triangle.	
Inside median		always Inside centroid	The centroid R of $\triangle ABC$ is two thirds of the distance from each vertex to the midpoint of the opposite side.	
Can be outside of altitude		Outside orthocenter	The lines containing the altitudes of $\triangle ABC$ are concurrent at the orthocenter S.	

HONORS MATH 2 CHAPTER LESSON 5.4

INDIRECT PROOF

New Vocabulary

- indirect reasoning assuming conclusion is false and then showing that the assumption was false (there was a counter example)
- indirect proof temp assume that what you are trying to prove is false then show your assumption is false ∴ the original thing is TRUE
- same proof by contradiction

KeyConcept How to Write an Indirect Proof

Step 1 Identify the conclusion you are asked to prove. Make the assumption that this conclusion is false by assuming that the opposite is true.

Step 2 Use logical reasoning to show that this assumption leads to a contradiction of the hypothesis, or some other fact, such as a definition, postulate, theorem, or corollary.

Step 3 Point out that since the assumption leads to a contradiction, the original conclusion, what you were asked to prove, must be true.

EXAMPLE 1 State the Assumption for Starting an Indirect Proof

A. State the assumption you would make to start an indirect proof for the statement $\overline{EF}$ *is not a perpendicular bisector.*

EF is a perp bisector

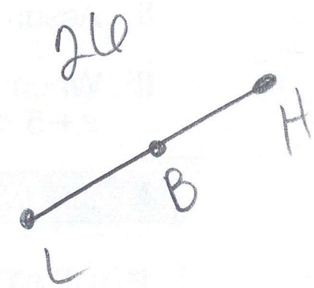
EXAMPLE 1 State the Assumption for Starting an Indirect Proof

B. State the assumption you would make to start an indirect proof for the statement $3x = 4y + 1$.

$3x \neq 4y + 1$

EXAMPLE 1 State the Assumption for Starting an Indirect Proof

C. State the assumption you would make to start an indirect proof for the statement *if* B *is the midpoint of* $\overline{LH}$ *and* $LH = 26$, *then* $\overline{BH}$ *is congruent to* $\overline{LB}$.



$\overline{BH}$ is NOT congruent to $\overline{LB}$

HONORS MATH 2 CHAPTER LESSON 5.4

INDIRECT PROOF

EXAMPLE 2 Write an Indirect Algebraic Proof

ao

Write an indirect proof to show that if $-2x + 11 < 7$, then $x > 2$.

Given: $-2x + 11 < 7$

Prove: $x > 2$

our assumption Must be false So

* The negation of $x > 2$ is $x \leq 2$ So
So assume $x = 2$ and $x < 2$ is TRUE
* what did you find →

$x < 2$ | $x = 2$
(1, 9) $9 < 7$ no | $7 < 7$?? No
(0, 11) $11 < 7$ no
(-1, 13) $13 < 7$ no

TRUE $x > 2$

x	$-2x + 11$
2	7
1	9
0	11
−1	13
−2	15

EXAMPLE 2 Check Your Progress

CheckPoint

Which is the correct order of steps for the following indirect proof?

Given: $x + 5 > 18$

Prove: $x > 13$

I. In both cases, the assumption leads to a contradiction. Therefore, the assumption $x \leq 13$ is false, so the original conclusion that $x > 13$ is true.

II. Assume $x \leq 13$.

III. When $x < 13$, $x + 5 = 18$ and when $x < 13$, $x + 5 < 18$.

A. I, II, III

B. I, III, II

C. II, III, I

D. III, II, I

Real-World Example 3 Indirect Algebraic Proof

EDUCATION Marta signed up for three classes at a community college for a little under \$156. There was an administration fee of \$15, and the class costs are equal. How can you show that each class cost less than \$47?

HONORS MATH 2 CHAPTER LESSON 5.4

INDIRECT PROOF

EXAMPLE 4 Indirect Proofs in Number Theory

Write an indirect proof to show that if x is a prime number not equal to 3, then $\frac{x}{3}$ is not an integer.

Given: x is prime not equal to 3

Prove: $\frac{x}{3}$ is Not an Integer

Step 1 Indirect: Assume $\frac{x}{3}$ is an Integer

$\frac{x}{3} = n \leftarrow \ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

Step 2: $\frac{x}{3} = n$ get x by itself

$x = 3n$

$x = 3$ times a # not 1 or 3 & prime

$x = 3 \cdot 7$ $\quad$ $x = 3 \cdot 11$

$x = 21$ $\quad$ $x = 33$

Neither of these are prime ∴

all theses # are Integers So Contradicts $\frac{x}{3}$ is an Integer

∴ $\frac{x}{3}$ is Not an Integer

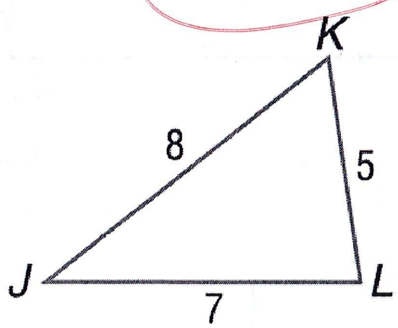
Get here

EXAMPLE 5 Geometry Proof

Write an indirect proof.

Given: ΔJKL with side lengths 5, 7, and 8 as shown.

Prove: $m\angle K < m\angle L$



Assume $m\angle K \geq m\angle L$

We know $JL \geq JK$ by angle-side Relation

$7 \geq 8$ false

Since Assumption leads to a Contradiction the assumption Must be false

∴ $m\angle K < m\angle L$

HONORS MATH 2 CHAPTER LESSON 5.3

INEQUALITIES IN ONE TRIANGLE

KeyConcept Definition of Inequality

Words	For any real numbers a and b, $a > b$ if and only if there is a positive number c such that $a = b + c$.
Example	If $5 = 2 + 3$, then $5 > 2$ and $5 > 3$.

if I tell you $6 = x + y$ what do you know about x : y compared to 6.

KeyConcept Properties of Inequality for Real Numbers

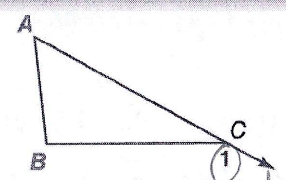
The following properties are true for any real numbers a, b, and c.

Comparison Property of Inequality	$a < b$, $a = b$, or $a > b$
Transitive Property of Inequality	1. If $a < b$ and $b < c$, then $a < c$. 2. If $a > b$ and $b > c$, then $a > c$.
Addition Property of Inequality	1. If $a > b$, then $a + c > b + c$. 2. If $a < b$, then $a + c < b + c$.
Subtraction Property of Inequality	1. If $a > b$, then $a - c > b - c$. 2. If $a < b$, then $a - c < b - c$.

Theorem 5.8 Exterior Angle Inequality

The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles.

Example: $m\angle 1 > m\angle A$
$m\angle 1 > m\angle B$



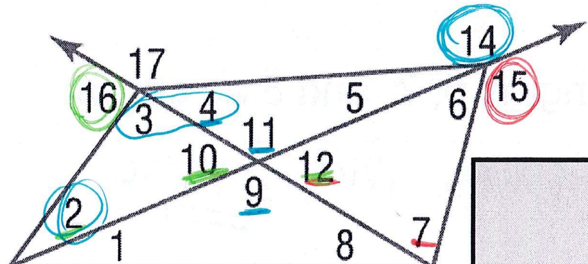
exterior ∠ of Δ has to be greater than then the other 2 angles

EXAMPLE 1 Use the Exterior Angle Inequality Theorem

A. Use the Exterior Angle Inequality Theorem to list all angles whose measures are less than $m\angle 14$.

m∠15 > m∠7
m∠15 > m∠12
m∠14 > m∠11 *
m∠14 > m∠4 *
m∠14 > m∠2
m∠14 > m∠9
m∠14 > m∠3 *
m∠14 > m∠3 + m∠4
m∠14 > m∠4

m∠16 > m∠2
m∠16 > m∠10



m∠11 = m∠9

m∠9 > m∠6
m∠9 > m∠7

m∠14 > m∠6
m∠14 > m∠7

B. Use the Exterior Angle Inequality Theorem to list all angles whose measures are greater than $m\angle 5$.

m∠10 > m∠5 ⇒ m∠5 < m∠10
m∠17 > m∠5 ⇒ m∠5 < m∠17
m∠12 > m∠5 ⇒ m∠5 < m∠12
m∠16 > m∠5 ⇒ m∠5 < m∠16

m∠5 < m∠12 vertical
m∠17 > m∠2 + m∠1

m∠17 > m∠5 + m∠6
m∠5 < m∠17

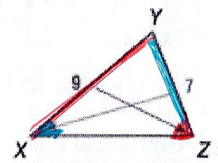
HONORS MATH 2 CHAPTER LESSON 5.3

INEQUALITIES IN ONE TRIANGLE

Theorems **Angle-Side Relationships in Triangles**

5.9 If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

Example: $XY > YZ$, so $m\angle Z > m\angle X$.

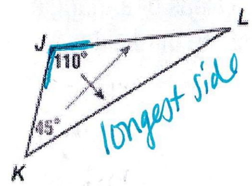


Angles opposite longest side is biggest

$m\angle Z > m\angle X$

5.10 If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

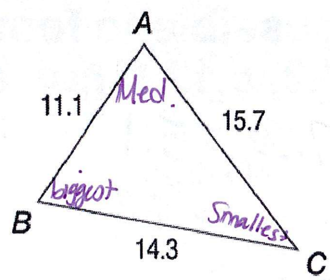
Example: $m\angle J > m\angle K$, so $KL > JL$.



$KL > KJ$

EXAMPLE 2 Order Triangle Angle Measures

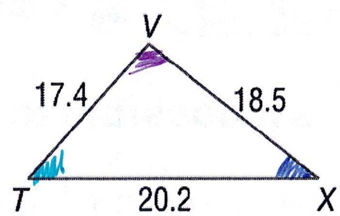
List the angles of ΔABC in order from smallest to largest.



$m\angle C < m\angle A < m\angle B$

EXAMPLE 2 Check Your Progress

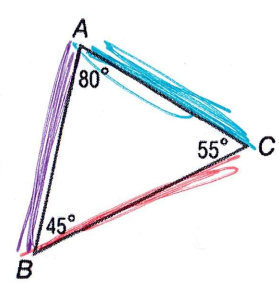
List the angles of ΔTVX in order from smallest to largest.



$m\angle X < m\angle T < m\angle V$

EXAMPLE 3 Order Triangle Side Lengths

List the sides of ΔABC in order from shortest to longest.



$AC < AB < BC$

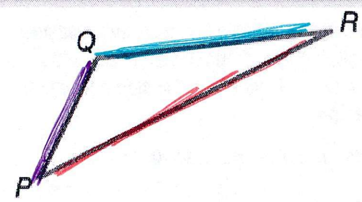
Sum 2 Sides always greater than 3rd Side

HONORS MATH 2 CHAPTER LESSON 5.5

TRIANGLE INEQUALITY

Theorem 5.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.



Examples $PQ + QR > PR$
$QR + PR > PQ$
$PR + PQ > QR$

EXAMPLE 1 Identify Possible Triangles Given Side Lengths

B. Is it possible to form a triangle with side lengths of 6.8, 7.2, 5.1? If not, explain why not.

$6.8 + 7.2 > 5.1$
$14 > 5.1$ ✓
$5.1 + 6.8 > 7.2$
$11.9 > 7.2$
$5.1 + 7.2 > 6.8$
$12.3 > 6.8$

Yes because Sum of 2 sides ALWAYS greater than 3rd Side

EXAMPLE 1 Check Your Progress

CheckPoint

A. Is it possible to form a triangle given the side lengths $8\frac{1}{2}$, $11\frac{1}{2}$, and $20\frac{1}{2}$?

$8\frac{1}{2} + 11\frac{1}{2} > 20\frac{1}{2}$
$20 > 20\frac{1}{2}$ nope
$20\frac{1}{2} + 8\frac{1}{2} > 11\frac{1}{2}$
$29 > 11\frac{1}{2}$ ✱
$20\frac{1}{2} + 11\frac{1}{2} > 8\frac{1}{2}$
$32 > 8\frac{1}{2}$

nope Cannot be a Δ because $8\frac{1}{2} + 11\frac{1}{2}$ is 20 and that isn't bigger than $20\frac{1}{2}$

HONORS MATH 2 CHAPTER LESSON 5.5

TRIANGLE INEQUALITY

STANDARDIZED TEST EXAMPLE 2 **Find Possible Side Lengths**

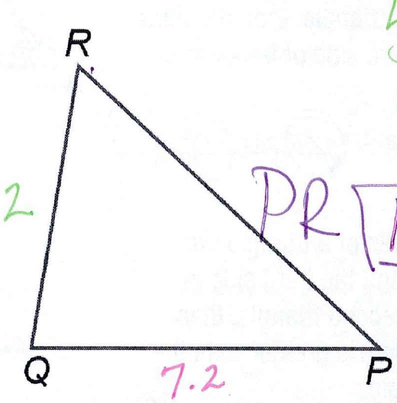
In ΔPQR, $PQ = 7.2$ and $QR = 5.2$. Which measure cannot be PR?

A 7

B 9

C 11

D 13



5.2

7.2

5.2

7.2

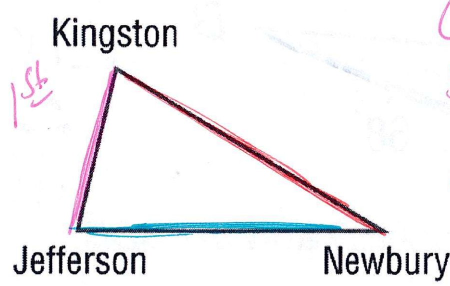
PR $\boxed{12.4}$ Must be greater than 12.4

$PR > 12.4$

EXAMPLE 3 **Proof Using Triangle Inequality Theorem**

TRAVEL The towns of Jefferson, Kingston, and Newbury are shown in the map below. Prove that driving first from Jefferson to Kingston and then Kingston to Newbury is a greater distance than driving from Jefferson to Newbury.

Because Δ Inequality thm

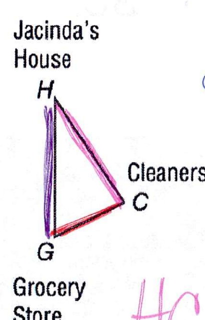


Creates a Δ So therefor

$KJ + KN > JN$

EXAMPLE 3 **Check Your Progress** CheckPoint

Jacinda is trying to run errands around town. She thinks it is a longer trip to drive to the cleaners and then to the grocery store, than to the grocery store alone. Determine whether Jacinda is right or wrong.



She is Correct because Δ

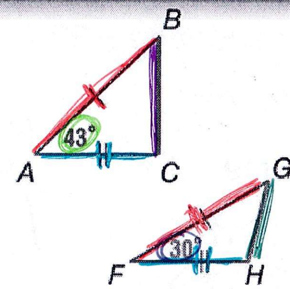
$HC + CG > HG$

HONORS MATH 2 CHAPTER LESSON 5.6

INEQUALITIES IN TWO TRIANGLES

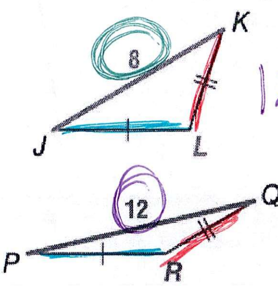
Theorems Inequalities in Two Triangles

5.13 Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.



Example: If $\overline{AB} \cong \overline{FG}$, $\overline{AC} \cong \overline{FH}$, and $m\angle A > m\angle F$, then $BC > GH$.

5.14 Converse of the Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the third side in the first is longer than the third side in the second triangle, then the included angle measure of the first triangle is greater than the included angle measure in the second triangle.

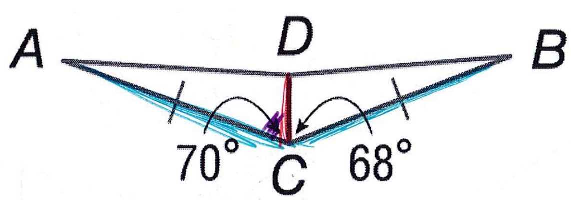


$12 > 8$

Example: If $\overline{JL} \cong \overline{PR}$, $\overline{KL} \cong \overline{QR}$, and $PQ > JK$, then $m\angle R > m\angle L$.

EXAMPLE 1 Use the Hinge Theorem and Its Converse

A. Compare the measures *AD* and *BD*.



$\overline{AC} \cong \overline{BC}$

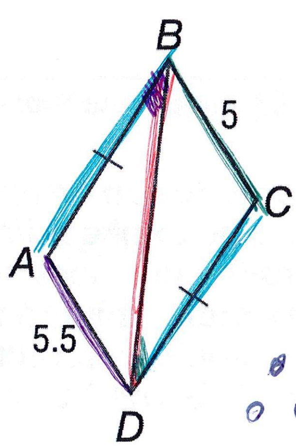
$\overline{DC} \cong \overline{DC}$ Reflexive

$\angle DCA > \angle DCB$

$\therefore AD > BD$ Hinge thm

B. Compare the measures $m\angle ABD$ and $m\angle BDC$.

In ΔABD and ΔBCD, $\overline{AB} \cong \overline{CD}$, $\overline{BD} \cong \overline{BD}$, and $AD > BC$.



$\overline{AB} \cong \overline{CD}$

$\overline{BD} \cong \overline{BD}$ Reflexive

$5.5 > 5$

$AD > BC$

$\therefore \angle ABD > \angle BDC$

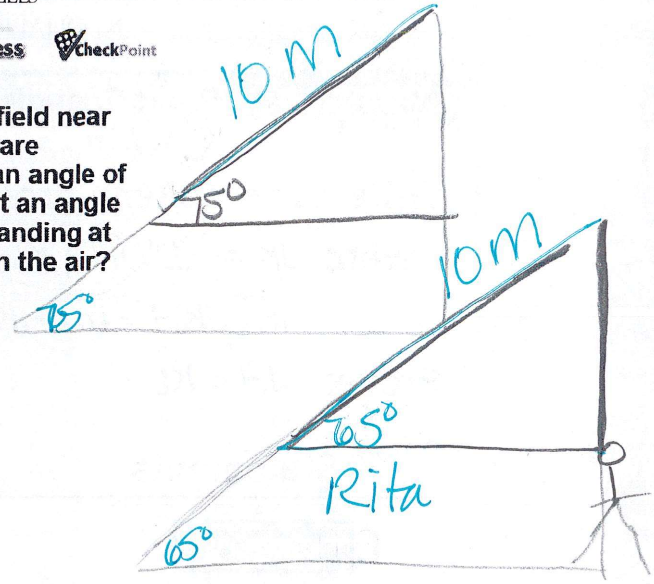
HONORS MATH 2 CHAPTER LESSON 5.6

Meena

INEQUALITIES IN TWO TRIANGLES

Real-World Example 2 **Check Your Progress** CheckPoint

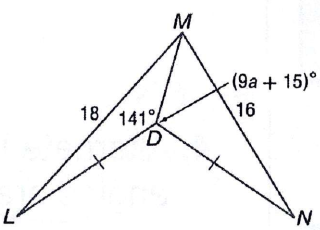
Meena and Rita are both flying kites in a field near their houses. Both are using strings that are 10 meters long. Meena's kite string is at an angle of 75° with the ground. Rita's kite string is at an angle of 65° with the ground. If they are both standing at the same elevation, which kite is higher in the air?



Meena's kite

EXAMPLE 3 **Apply Algebra to the Relationships in Triangles**

ALGEBRA Find the range of possible values for *a*.



$9a + 15 < 141$
$-15 \quad -15$
$9a < 126$
$\frac{9a}{9} < \frac{126}{9}$
$a < 14$

$9a + 15 > 0$
$-15 \quad -15$
$9a > -15$
$a > -\frac{5}{3}$

From the diagram we know that $\overline{LD} \cong \overline{ND}$, $\overline{MD} \cong \overline{MD}$, and $ML > MN$.

Must be tween

$-\frac{5}{3} < a < 14$

EXAMPLE 3 **Check Your Progress** CheckPoint

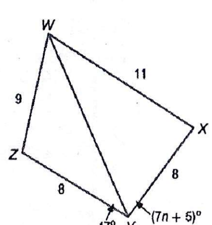
Find the range of possible values of *n*.

A. $6 < n < 25$

B. $\frac{52}{7} < n < \frac{175}{7}$

C. $n > 6$

D. $6 < n < 18.3$



$7n + 5 > 47$
$-5 \quad -5$
$\frac{7n}{7} > \frac{42}{7}$
$n > 6$

$7n + 5 > 180$
$-5 \quad -5$
$\frac{7n}{7} > \frac{175}{7}$
$n > 25$

can't be bigger than 180°

remember $11 + 8 = ?$

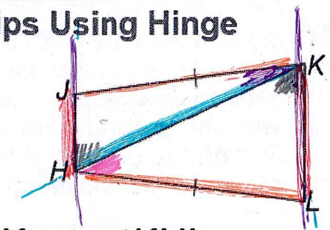
$6 < n < 25$

n 17

HONORS MATH 2 CHAPTER LESSON 5.6

INEQUALITIES IN TWO TRIANGLES

EXAMPLE 4 Prove Triangle Relationships Using Hinge Theorem



Write a two-column proof.

Given: $JK = HL$; $\overline{JH} \parallel \overline{KL}$

$m\angle JKH + m\angle HKL < m\angle JHK + m\angle KHL$

Prove: $JH < KL$

Statements	Reasons
1. JK = HL	1. Given
2. $HK = HK$	2. Reflexive Property
3. m∠JKH + m∠HKL < m∠JHK + m∠KHL	3. Given
JH ∥ KL 4. m∠HKL ≅ m∠JHK	Given 4. Alternate Interior angles are ≅
5. m∠JHK + m∠JKH < ~~m∠JHK~~ + m∠KHL	5. Substitution
6. $m\angle JKH < m\angle KHL$ Purple Smaller pink	6. Subtraction
7. JH < KL	7. Hinge Theorem

EXAMPLE 5 Prove Relationships Using Converse of Hinge Theorem

tough one

SSS

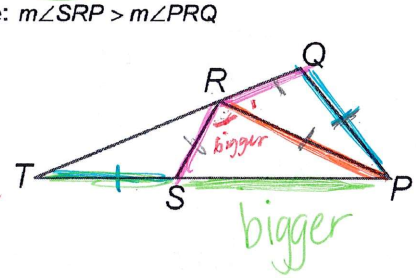
throw that in there

Given: $ST = PQ$

$SR = QR$

$ST = \frac{2}{3}SP$

Prove: $m\angle SRP > m\angle PRQ$



EXAMPLE 5 Prove Relationships Using Converse of Hinge Theorem

Answer:

Proof:

Statements	Reasons
1. ST = PQ	1. Given
2. PR = PR	2. Reflexive
3. SR = QR	3. Given
4. ST = ⅔SP SP > ST	4. Given
5. SP > PQ	5. Substitute
6. m∠SRP > m∠PRQ	6. SSS

Hard one