Pg. 464 9, 13, 15, 26, 27, 45 pg. 520 3, 9, 15, 17, 26 29

Name: _____

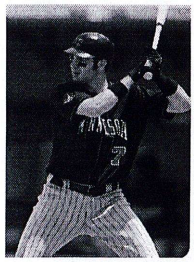
Honors Math 2 Chapter 7 Notes

7.1 and 7.7 - Ratios, Proportions, and Scale Drawings

Wed

A ratio is a comparison of two quantities using division. Ratios are expressed in simplest form. (reduced).

Two ratios with equal values are equivalent.



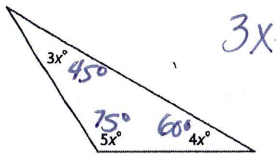
A baseball player's batting average is the ratio of the number of base hits to the number of at-bats, not including walks. Minnesota Twins' Joe Mauer had the highest batting average in the MLB in 2006 (old book...). If he had 521 official at bats and 181 hits, what is his batting average?

total 521 base hits / #at bats $\frac{181}{521} = .347$

Extended Ratios can be used to compare three or more quantities. The expression *a:b:c* means that the ratio of the 1st value to the 2nd is *a:b*, the 2nd to the 3rd is *b:c*, and the 1st to the 3rd is *a:c*.

EXAMPLE:

The ratio of the measures of the angles in a triangle is *3:4:5*.
Find the measures of the angles: 45° 60° 75° (3 : 4 : 5)



$3x+4x+5x=180^\circ$
$12x=180$
$x=15$

An equation stating that two ratios are equal is called a Proportion.

extreme → $\frac{a}{b} = \frac{c}{d}$ ← mean
mean → ← extreme

The _____ states that the product of the **means** is equal to the product of the **extremes**.

Solve the proportions:

$\frac{x}{4} = \frac{11}{-6}$ $-6x = 44$ $x = \frac{-22}{3} = 7.3$

$-\frac{4}{7} = \frac{6}{2y+5}$ $-\frac{4}{7} = \frac{6}{-10.5}$ $2y+5 = -10.5$ $2y = -15.5$ $y = -7.75$

$\frac{7}{z-1} = \frac{9}{z+4}$ $7(z+4) = 9(z-1)$ $7z+28 = 9z-9$ $-7z+9 \quad -7z+9$ $\frac{37}{2} = \frac{2z}{2}$ $18.5 = z$

$\frac{-4(2y+5)}{-4} = \frac{42}{-4}$

A **scale drawing or model** is a drawing/model with lengths proportional to the object it represents. The **scale** is the ratio of the lengths in the scale drawing to the real object.

Hot Wheels™ cars are *typically* 1:64 scale. If the Hot Wheels™ F-150 is 3.9" long, how long is the real thing?

$\frac{1}{64} = \frac{3.9}{x}$ $x = 249.6$ inches = 20.8 ft long

Suppose you want to build a model of the St. Louis Gateway Arch that is no more than 11" tall. Choose an appropriate scale and use it to determine the height of the model. The actual arch is 630' tall.

$\frac{11}{11} = \frac{630}{11} = \frac{x}{11} = 7560$

***Bonus:** To whom is the Gateway Arch a monument?*

$\frac{11}{7560}$ $\frac{}{11}$

7.2 -Similar Polygons Thurs pg. 473 10, 11, 13, 15, 35, 36, 51, 56, 59

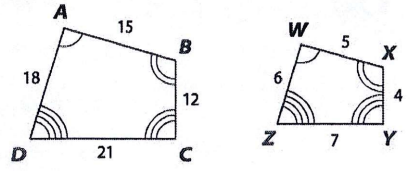
Two polygons are congruent IFF every corresponding measurement is congruent between the two polygons.

(IFF = _____)

KeyConcept Similar Polygons

Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

Example In the diagram below, *ABCD* is similar to *WXYZ*.



Corresponding angles

$\angle A \cong \angle W, \angle B \cong \angle X, \angle C \cong \angle Y,$ and $\angle D \cong \angle Z$

Corresponding sides

$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{3}{1}$

Symbols *ABCD* ~ *WXYZ*

$\Delta ABC \cong \Delta XYZ$

Triangles ABC and XYZ are congruent - they are exactly the same shape **and** size.

$\Delta ABC \sim \Delta XYZ$

Triangles ABC and XYZ are **similar** – they are exactly the same shape, but not the same size

If $\Delta EGH \sim \Delta IKL$, list all pairs of congruent angles, and write a proportion that relates the corresponding sides.

$\angle E \cong \angle J$
$\angle G \cong \angle K$
$\angle H \cong \angle L$

$\frac{FG}{JK} = \frac{GH}{KL}$ $\frac{FH}{JL}$

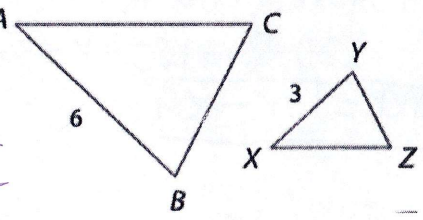
The ratio of the lengths of the corresponding sides of the similar triangles is the _____.

In the diagram, $\Delta ABC \sim \Delta XYZ$. The **scale factor** of ΔABC *to* ΔXYZ *is* —— *or*

The **scale factor** of ΔXYZ *to* ΔABC *is* —— *or*

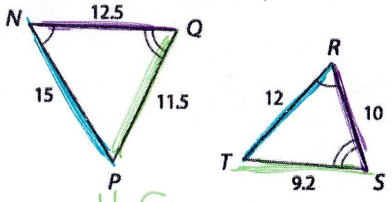
$\frac{6}{3} = 2$

$\frac{3}{6} = \frac{1}{2}$



Determine whether the triangles shown are similar. if so, write the similarity statement and scale factor:

yes $\Delta NQP \sim \Delta$ RST

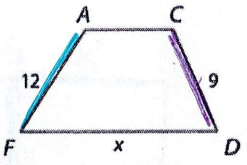


$\frac{12.5}{10} = 1\frac{1}{4} = \frac{5}{4}$

$\frac{15}{12} = \frac{5}{4}$ ✓

$\frac{11.5}{9.2} = \frac{5}{4}$ ✓

In the figures, ACDE ~ VWYZ
Find *x* and *y*:



$\frac{9}{6} = \frac{12}{3y-1}$

$27y - 9 = 72$

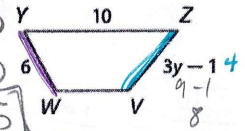
$27y = 81$

$y = 3$

$\frac{9}{6} = \frac{x}{10}$

$6x = 90$

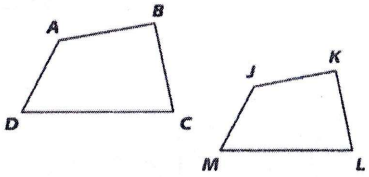
$x = 15$



4
9-1
8

30 Units

If two polygons are similar, then their perimeters are proportional to the scale factor betwixt them.

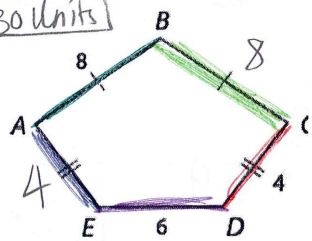


Example If *ABCD* ~ *JKLM*, then

$\frac{AB + BC + CD + DA}{JK + KL + LM + MJ} = \frac{AB}{JK} = \frac{BC}{KL} = \frac{CD}{LM} = \frac{DA}{MJ}$.

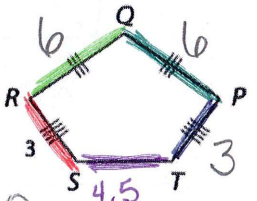
ABCDE ~ *PQRST*. Find the scale factor of *ABCDE to PQRST* and the perimeter of each polygon:

Scale factor $\frac{4}{3}$



$\frac{4}{3} = \frac{6}{x}$

$4x = 18$



$\frac{4}{3} = \frac{8}{x}$

$4x = 24$

$\frac{4}{3} = \frac{30}{P}$

$4P = 90$

$P = 22.5$ units

$\frac{22.5}{2}$

ACT – Type Question: Friday

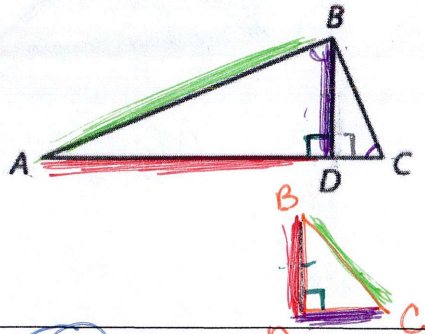
In the figure, $\angle ADB$ is a right angle. Which of the following would *not* be sufficient to prove that $\triangle ADB \sim \triangle CDB$?

A $\frac{AD}{BD} = \frac{BD}{CD}$ SAS

B $\frac{AB}{BC} = \frac{BD}{CD}$

C $\angle ABD \cong \angle C$ AAA

D $\frac{AD}{BD} = \frac{BD}{CD} = \frac{AB}{BC}$ SSS

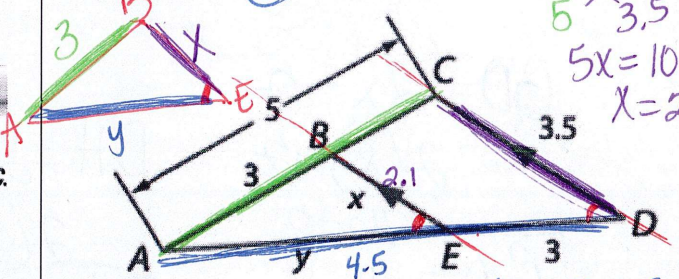


Like **equality** and **congruency**, **similarity** is *reflexive*, *symmetric*, and *transitive*!

Theorem 7.4 Properties of Similarity

Reflexive Property of Similarity	$\triangle ABC \sim \triangle ABC$
Symmetric Property of Similarity	If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.
Transitive Property of Similarity	If $\triangle ABC \sim \triangle DEF$, and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.

Find BE and AD: 2.1 7.5



$\frac{3}{5} = \frac{x}{3.5}$

$5x = 10.5$

$x = 2.1$

$\frac{3}{5} = \frac{y}{y+3}$

$5y = 3y + 9$

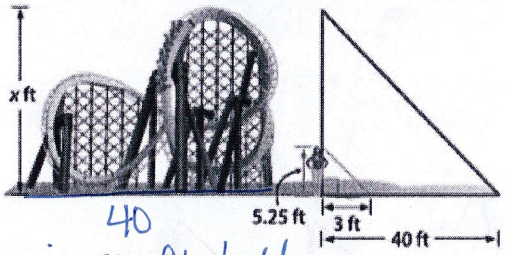
$-3y \quad -3y$

$2y = 9$

$y = 4.5$

ROLLER COASTERS **Hallie is estimating the height of the Superman roller coaster in Mitchellville, Maryland. She is 5 feet 3 inches tall and her shadow is 3 feet long. If the length of the shadow of the roller coaster is 40 feet, how tall is the roller coaster?**

Understand Make a sketch of the situation. 5 feet 3 inches is equivalent to 5.25 feet.

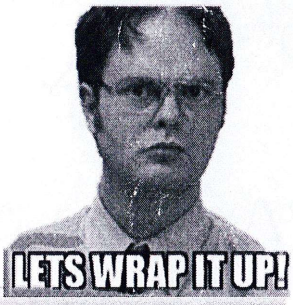


$\frac{5.25}{3} = \frac{x}{40}$

$3x = 210$

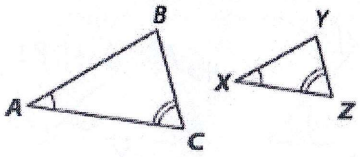
$x = 70$

The roller coaster is 70 ft tall



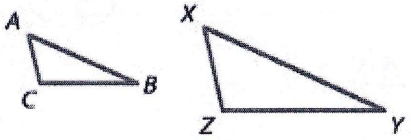
ConceptSummary Triangle Similarity

AA Similarity Postulate



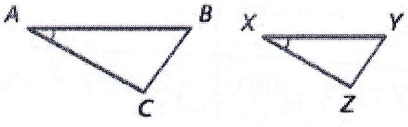
If $\angle A \cong \angle X$ and $\angle C \cong \angle Z$, then $\triangle ABC \sim \triangle XYZ$.

SSS Similarity Theorem



If $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

SAS Similarity Theorem



If $\angle A \cong \angle X$ and $\frac{AB}{XY} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

7.3 - Similar Triangles

pg. 484 9-14, 20-22, 27, 39, 44

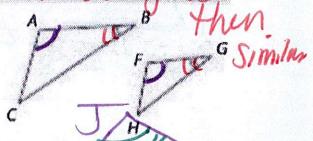
We have several congruency postulates for triangles: SSS, SAS, ASA, and AAS. There are also similarity postulates for triangles!

*If two angles of one triangle are congruent to two angles of another triangle, then the third angles **must** also be ≅ by the Third Angle Theorem.* So really this is AAA similarity

Postulate 7.1 Angle-Angle (AA) Similarity

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

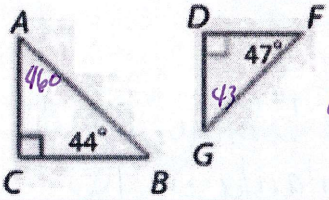
Example If $\angle A \cong \angle F$ and $\angle B \cong \angle G$, then $\triangle ABC \sim \triangle FGH$.

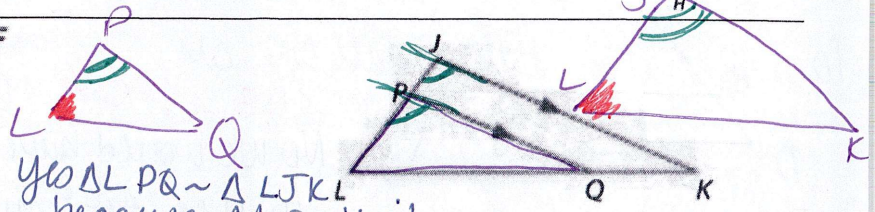


If 2 angles ≅ then Similar

Are these triangles similar?

No because all angles are not ≅





yes $\triangle LPQ \sim \triangle LJKL$ because AA Similarity

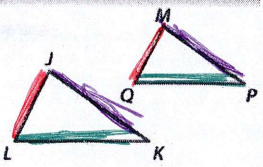
We also have SSS and SAS Similarity Postulates, but there is an important difference:

Theorems Triangle Similarity

7.2 Side-Side-Side (SSS) Similarity

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

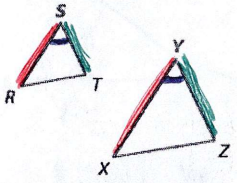
Example If $\frac{JK}{MP} = \frac{KL}{PQ} = \frac{LJ}{QM}$, then $\triangle JKL \sim \triangle MPQ$.



7.3 Side-Angle-Side (SAS) Similarity

If the lengths of two sides of one triangle are proportional to the lengths of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

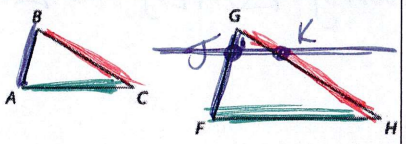
Example If $\frac{RS}{UV} = \frac{ST}{VW}$ and $\angle S \cong \angle Y$, then



Proof Theorem 7.2

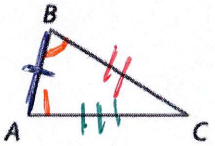
Given: $\frac{AB}{FG} = \frac{BC}{GH} = \frac{AC}{FH}$

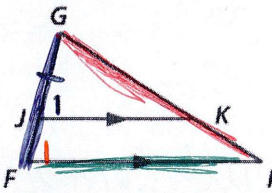
Prove: $\triangle ABC \sim \triangle FGH$

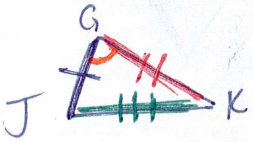


Paragraph Proof:

Locate J on $\overline{FG}$ so that $JG = AB$.
Draw $\overline{JK}$ so that $\overline{JK} \parallel \overline{FH}$.
Label $\angle GJK$ as $\angle 1$.







Since $\angle G \cong \angle G$ by the Reflexive Property and $\angle 1 \cong \angle F$ by the Corresponding Angles Postulate, $\triangle GJK \sim \triangle GFH$ by the AA Similarity Postulate.

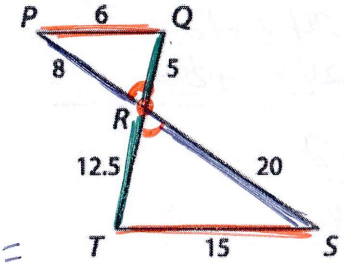
By the definition of similar polygons, $\frac{JG}{FG} = \frac{GK}{GH} = \frac{JK}{FH}$. By substitution, $\frac{AB}{FG} = \frac{GK}{GH} = \frac{JK}{FH}$.

Since we are also given that $\frac{AB}{FG} = \frac{BC}{GH} = \frac{AC}{FH}$, we can say that $\frac{GK}{GH} = \frac{BC}{GH}$ and $\frac{JK}{FH} = \frac{AC}{FH}$. This means that $GK = BC$ and $JK = AC$, so $\overline{GK} \cong \overline{BC}$ and $\overline{JK} \cong \overline{AC}$.

By SSS, $\triangle ABC \cong \triangle JGK$.

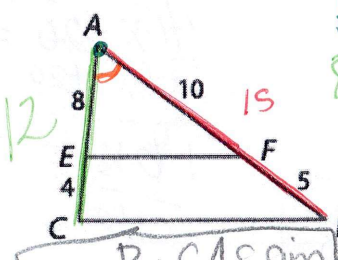
By CPCTC, $\angle B \cong \angle G$ and $\angle A \cong \angle 1$. Since $\angle 1 \cong \angle F$, $\angle A \cong \angle F$ by the Transitive Property. By AA Similarity, $\triangle ABC \sim \triangle FGH$.

Determine whether the triangles in each image are similar. If they are, write a similarity statement. If not, explain why not.



$\frac{6}{15} = \frac{2}{5} = \frac{5}{12.5} = \frac{8}{20}$ ✓ ✓ ✓

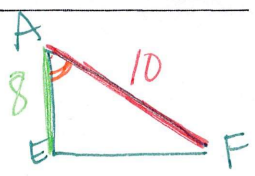
yes $\triangle QRP \sim \triangle TRS$



By SAS sim
yes $\triangle ABC \sim \triangle AFE$

$\frac{12}{8} = \frac{15}{10}$

$\frac{3}{2} = \frac{3}{2}$



#1

Solve each proportion.

a. $\frac{6x}{27} = \frac{43}{1}$

$1161 = 6x$

$x = 193.5$

b. $\frac{3x-5}{4} = \frac{-5}{7}$

$-20 = 21x - 35$

$+35 \quad +35$

$\frac{-15}{21} = \frac{21x}{21}$

$-\frac{5}{7} = x$

$-.71$

#2

a. The ratio of measures of the sides of a triangle is 6:7:9 the perimeter is 77 cm. Find the measures of each side of the triangle.

$6x + 7x + 9x = 77$

$22x = 77$

$x = 3.5$

$21 \times 24.5 \times 31.5$

21 cm 24.5 cm 31.5 cm

b. The ratio of goats to sheep at a university research farm is 4:7. The number of sheep at the farm is 28. How many goats are there?

$\frac{4 \text{ goats}}{7 \text{ sheep}}$ $\frac{4}{7} = \frac{x}{28}$ $x = 16$ goats

#3

MONA LISA A visitor to the Louvre Museum in Paris wants to sketch a drawing of the Mona Lisa, a famous painting. The original painting is 77 centimeters by 53 centimeters. Choose an appropriate scale for the replica so that it will fit on a 8.5-by-11-inch sheet of paper.

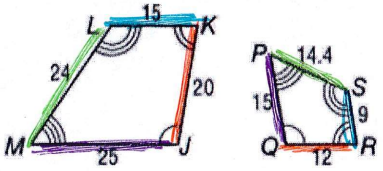
77 cm x 53 cm

11 inches x 8.5 inches

$\frac{11}{77}$ 1 in for 7 cm

#4

Determine whether each pair of figures is similar. If so, write the similarity statement and the scale factor.



$\frac{15}{9} = \frac{20}{12} = \frac{24}{14.4} = \frac{25}{15}$

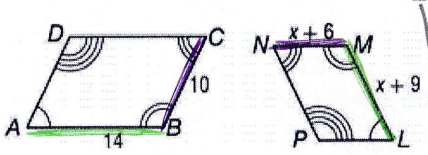
$\frac{5}{3} = \frac{5}{3} = \frac{5}{3}$ scale factor

JKLM ~ QRSP

#5

Each pair of polygons is similar. Find the value of *x*.

a.



$X = \frac{3}{2}$

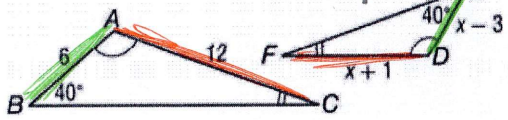
$\frac{10}{x+6} = \frac{14}{x+9}$

$14x + 84 = 10x + 90$

$-10x \quad -84 \quad -10x \quad -84$

$4x = 6$

b.



$\frac{12}{x+1} = \frac{6}{x-3}$

$6x + 6 = 12x - 36$

$-6x \quad -6x$

$42 = 6x$

$6 = 6x - 36$

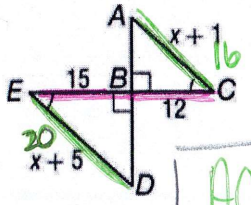
$+36 \quad +36$

$X = 7$

#6

Identify the similar triangles. Then find each measure.

AC



$AC = 16$

$\triangle ABC \sim \triangle DBE$

$\frac{15}{12} = \frac{x+5}{x+1}$

$12x + 60 = 15x + 15$

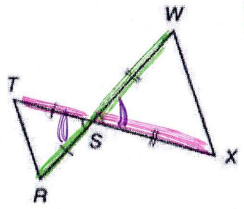
$-12x \quad -15 \quad -12x \quad -15$

$45 = 3x$

$15 = x$

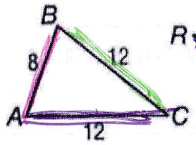
#7

Determine whether each pair of triangles is similar. If so, write the similarity. If not explain why not.



yes $\triangle SRT \sim \triangle SWX$

SAS Similarity



$\frac{9}{12} = \frac{9}{12} = \frac{6}{8}$

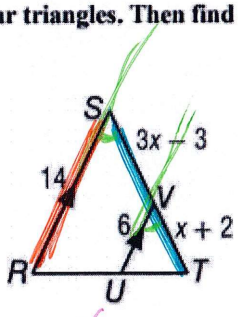
yes Scale factor $\frac{3}{4}$ or $\frac{4}{3}$

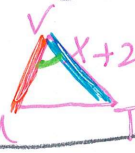
$\triangle ABC \sim \triangle PQR$

SSS Similarity

#8

Identify the similar triangles. Then find each measure.





yes Similar SAS

$\triangle RST \sim \triangle UVT$ SS

$\frac{3x-3+x+2}{x+2} = \frac{14}{6}$ (4x−1)

$14x + 28 = 24x - 6$

$-14x \quad +6 \quad -14x \quad +6$

$34 = 10x$

$3.4 = x$

$VT = 3.4 + 2$

$VT = 5.4$

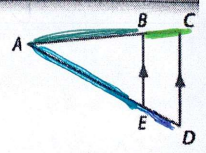
11/26/18

7.4 – Parallel Lines and Proportional Parts

Proportional Parts Within Triangles When a triangle contains a line that is parallel to one of its sides, the two triangles formed can be proved similar using the Angle-Angle Similarity Postulate. Since the triangles are similar, their sides are proportional.

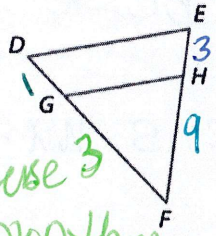
Theorem 7.5 Triangle Proportionality Theorem (Converse)

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.



Example If $\overline{BE} \parallel \overline{CD}$, then $\frac{AB}{BC} = \frac{AE}{ED}$

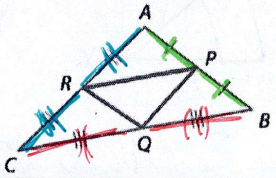
In ΔDEF, $EH = 3$, $HF = 9$, and DG is one-third the length of GF. Is $\overline{DE} \cong \overline{GH}$?



$\frac{1}{3} = \frac{3}{9}$ Converse 3

yes because Δ prop thm

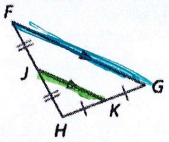
A Midsegment of a triangle is a segment with Endpoints that are the midpoints of two sides of the triangle. Every triangle has three midsegments.



Theorem 7.7 Triangle Midsegment Theorem

A midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.

Example If J and K are midpoints of $\overline{FH}$ and $\overline{HG}$, respectively, then $\overline{JK} \parallel \overline{FG}$ and $JK = \frac{1}{2}FG$.



$JK = \frac{1}{2}FG$

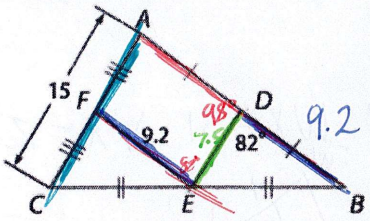
Find each measure:

$DE = \frac{1}{2}(15) = 7.5$

$DB = 9.2$

$m\angle FED = 82°$

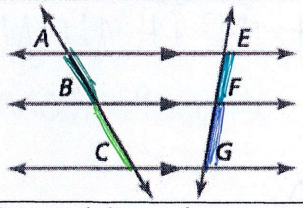
$9.2 = \frac{1}{2}x$

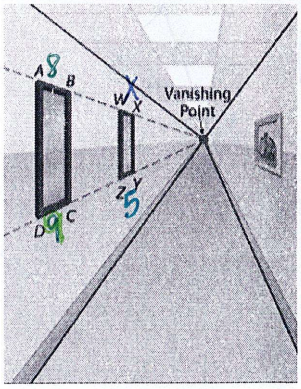


NOTES

If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, then $\frac{AB}{BC} = \frac{EF}{FG}$





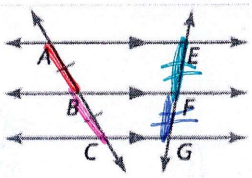
Megan is drawing a hallway in one-point perspective. She uses the guidelines shown to draw two windows on the left wall. If segments AD, BC, WZ, and XY are all parallel, $AB = 8cm$, $DC = 9cm$, and $ZY = 5cm$, find the length of WX:

$\frac{8}{9} = \frac{x}{5}$ $\frac{9x}{9} = \frac{40}{9}$ $WX = 4.4\,cm$

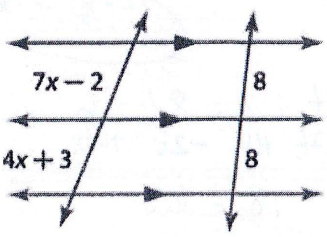
More NOTES

If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, and $\overline{AB} \cong \overline{BC}$, then $\overline{EF} \cong \overline{FG}$.



Find x:



$7x - 2 = 4x + 3$

$-4x \quad +2 \quad -4x \quad +2$

$3x = 5$

$x = \frac{5}{3}$

7.5 – Parts of Similar Triangles Tuesday 11/27/18

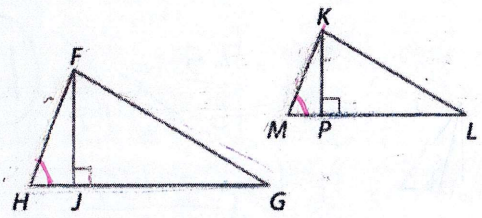
We know that corresponding side lengths are proportional in similar triangles, but it goes further than that:

- Altitudes
- Angle bisectors
- and Medians

} Are also all proportional between similar triangles!

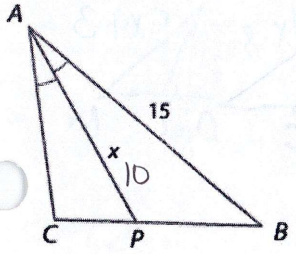
Given: $\triangle FGH \sim \triangle KLM$
$\overline{FJ}$ and $\overline{KP}$ are altitudes.

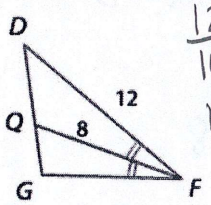
Prove: $\frac{FJ}{KP} = \frac{HF}{MK}$



1. $\triangle FGH \sim \triangle KLM$ Given
2. $\angle H \cong \angle M$ 2) Def Similar $\triangle$
3. $\angle FJH \cong \angle KPM$ 3) Both right $\angle$'s
4. $\triangle FJH \sim \triangle KPM$ 4) AA $\triangle$ Similarity
5. $\frac{FJ}{KP} = \frac{HF}{MK}$ 5) Def of Similar $\triangle$'s

In the figure, $\triangle ABC \sim \triangle FDG$. Find the value of x:





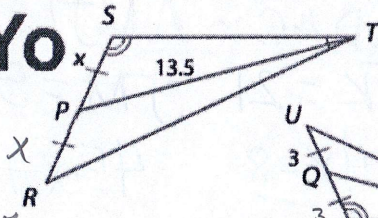
$\frac{12}{15} = \frac{8}{x}$

$12x = 120$

$x = 10$

Find the value of x:

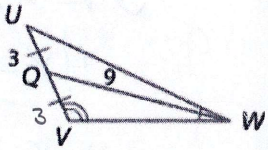
YoYo



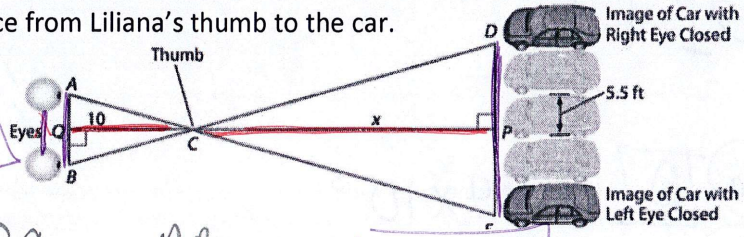
$\frac{2x}{6} = \frac{13.5}{9}$

$18x = 81$

$x = 4.5$



Liliana holds her arm straight out in front of her with her elbow straight and thumb pointed up. Closing one eye, she aligns one edge of her thumb with a car she is sighting. Next she switches eyes without moving her head or arm. The car appears to jump 4 car widths. If Liliana's thumb is about 10 times farther from her eye than the distance between her eyes, and the car is about 5.5 feet wide, estimate the distance from Liliana's thumb to the car.



Distance from thumb to car = X ft

$\frac{10}{1} = \frac{x}{22}$

$x = 220$

220 ft

$\frac{PC}{QC} = \frac{AB}{DF}$

The distance between your eyes = 1 = a

The distance from your thumb to your eye = 10 = b

Ratio of your a/b = $\frac{1}{10}$ or $\frac{10}{1}$

$\frac{10}{x} = \frac{1}{5.5 \cdot 4}$

$\frac{10}{x} = \frac{1}{22}$

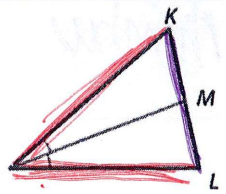
YoYo

Now you can use this method to estimate distance!

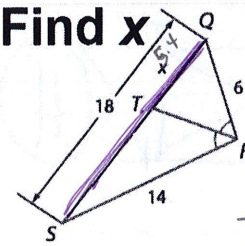
Theorem 7.11 Triangle Angle Bisector

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

Example If $\overline{JM}$ is an angle bisector of $\triangle JKL$,
then $\frac{KM}{LM} = \frac{KJ}{LJ}$ ← segments with vertex K / ← segments with vertex L



Find x



$\frac{18-x}{x} = \frac{14}{6}$

$14x = 108 - 6x$

$+6x \quad +6x$

$20x = 108$

$x = 5.4$