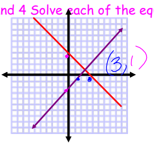
College Algebra Chapter 5
5-1 Linear and 5-4 nonlinear systems of equations

System of equation: equations that involve 2 or more variables
Solution of a system: Is an ordered pair that satisfies each equation in the system
Solving the system: finding the set of all solutions

In this chapter we will learn 4 ways to solve systems
1. Substitution
2. Graphing
3. Elimination
4. Solving by matrices GOODY
5. Gaussian elimination

Example 1, 3 and 4 Solve each of the equations by substitution

$x+y=4$
$x-y=2$



(3, 1)

$x^2+4x-y=7$
$2x-y=-1$ +y +y

(-4, -7)
(2, 5)

$x+y=-4$
$x^2+y=3$

$y=x^2+4x-7$
$2x-(x^2+4x-7)=-1$
$2x-x^2-4x+7=-1$
$-x^2-2x+8=0$
$x^2+2x-8=0$
$(x+4)(x-2)=0$
$x=-4$ $x=2$

-8
4, -2 2

$-8-y=-1$
$+8$ $+8$
$-y=7$
$2(2)-y=-1$ $4-y=-1$ -4 $-y=-5$

Example 2 Solving systems by substitution
A total of $12,000 is invested in two funds paying 5% and 3% simple interest. (I=prt)
The yearly interest is $500. How much is invested at each rate?

$I=P_1rt$
5%

$\boxed{X = \text{Amount Invested}}$

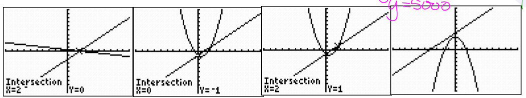
$500=X\cdot.05\cdot1$
$X=10,000$

$I=P_2rt$
3%
$500=y(.03)(1)$
$16,666=y

$X(.05)+.03y=500$
$X+y=12000$
$-y$ $-y$
$X=12,000-y$

$(12,000-y)(.05)+.03y=500$
$600-.05y+.03y=500$
$600-.02y=500$
-600 -600
$-.02y=-100$
$y=5000$

Solving by Graphing
Solution is the points of intersection(s)



→ on Calc you Must get y by itself

Example 5 solve the system by graphing

$y=\ln x$
$x+y=1$
$y=-x+1$

$x+y=4$
$x^2+y^2-4x=0$

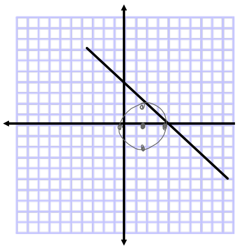
$(x-h)^2+(y-k)^2=r^2$

$x^2-4x+\boxed{4}+y^2=0+4$
$(x-2)^2+y^2=4$
(2, 0)
$r=2$

$y=-x+4$
$x^2+(-x+4)^2-4x=0$
$x^2+x^2-8x+16-4x=0$
$2x^2-12x+16=0$
$2(x^2-6x+8)=0$
$2(x-4)(x-2)=0$
$x=4$ $x=2$

8
-4 -2 -6

$x=4$ $y=0$
$x=2$ $y=2$



Example 6 BREAK EVEN ANALYSIS
A shoe company invests $300,000 in equipment to produce a new line of athletic footwear. Each pair of shoes cost $5 to produce and is sold for $60. How many pairs of shoes must be sold before the business breaks even?

Example 7 Movie ticket sales
The weekly ticket sales for a new comedy movie decreased each week. At the same time, the weekly ticket sales for a new drama movie increased each week. Models that approximate the weekly ticket sales S (in millions of dollars) for each movie are
Where x represents the number of weeks each movie was in the theaters, with x=0 corresponding to the ticket sales during the opening weekend. After how many weeks will the ticket sales for the two movies be equal?

$S=60-8x$
$S=10+4.5x$

$\sqrt{y^2}=\sqrt{-2x^2+18}$
$y=\sqrt{-2x^2+18}$
$y=-\sqrt{-2x^2+18}$

$2x^2+y^2=18$
$\frac{xy}{x}=\frac{4}{x}$
$y=\frac{4}{x}$

$2x^2+\left(\frac{4}{x}\right)^2=18$
$2x^2+\frac{16}{x^2}=18$

pg. 367 1-51 EOO

College Algebra 5-1
Solving by elimination

Date: 12/9/13

I. The Method of Elimination (Pages 465–467)

The operations that can be performed on a system of linear equations to produce an equivalent system are:

(1) interchanging any two equations

(2) multiplying an equation by a nonzero constant

(3) adding a multiple of one equation to any other equation in the system

List the steps necessary for solving a system of equations using the method of elimination.

1. Obtain coefficients for x (or y) that differ only in sign by multiplying all terms of one or both equations by suitably chosen constants.
2. Add the equations to eliminate one variable and solve the resulting equation.
3. Back-substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.
4. Check your solution in both of the original equations.

opposite

$2x + -2x = 0 \quad -6y + 6y = 0$ Create opposites

Example 1, 2 and 3 Solve each system by Elimination

$+\begin{cases} 3x + 2y = 4 \\ 5x - 2y = -1 \end{cases}$ $\left(\frac{3}{8}, \frac{23}{16}\right)$

$\frac{8x}{8} = \frac{3}{8}$

$x = \frac{3}{8}$

$3\left(\frac{3}{8}\right) + 2y = 4$

$\frac{9}{8} + 2y = \frac{32}{8}$ $-\frac{9}{8}$ $-\frac{9}{8}$

$2y = \frac{23}{8}$

$\frac{1}{2} \cdot 2y = \frac{23}{8} \cdot \frac{1}{2}$

$y = \frac{23}{16}$

$\begin{cases} 2x - 3y = -7 \\ 3(3x + y) = (-5)3 \end{cases}$ $(-2, 1)$

$2x - 3y = -7$

$+\ 9x + 3y = -15$

$\frac{11x}{11} = \frac{-22}{11}$

$x = -2$

$3(-2) + y = -5$

$-6 + y = -5$

$+6 \qquad +6$

$\begin{cases} 4(5x + 3y) = (9)4 \\ 3(2x - 4y) = (14)3 \end{cases}$ $(3, -2)$

$20x + 12y = 36$

$+\ 6x - 12y = 42$

$\frac{26x}{26} = \frac{78}{26}$

$x = 3$

$5(3) + 3y = 9$

$15 + 3y = 9$

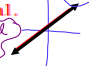
$-15 \qquad -15$

$3y = -6$

If a system of linear equations has two different solutions, it must have an infinite number of solutions.

For a system of two linear equations in two variables, list the possible number of solutions the system can have and give a graphical interpretation of the solutions.

1. Exactly one solution—the two lines intersect at one point. Different Slopes
2. Infinitely many solutions—the two lines are identical. Same Slope Same y-intercept
3. No solution—the two lines are parallel. Same Slope but different y-inter



If a contradictory statement such as $9 = 0$ is obtained while solving a system of linear equations using the method of elimination, then the system has no solution. False

If a statement that is true for all values of the variable, such as $0 = 0$, is obtained while solving a system of linear equations using the method of elimination, then the system has infinitely many solutions. $7=7$

$Ax + By = C$ $\quad m = -\frac{A}{B}$

$y = mx + b$ $\quad m = \text{Slope}$

Example 4 find the number of solutions by just looking at the slope and the y-intercept

$\begin{cases} \#1\ 2x - 3y = 3 \\ \#2\ -4x + 6y = 6 \end{cases}$

$\#1\ m = -\frac{2}{-3} = \frac{2}{3}$ Cover up Method $b = -1$

$\#2\ m = \frac{4}{6} = \frac{2}{3}$ $b = 1$

No Solutions

$\begin{cases} 2x - 3y = 3 \\ x + 2y = 5 \end{cases}$

$m = -\frac{2}{-3} = \frac{2}{3}$

$m = -\frac{1}{2}$

One Solution

$\begin{cases} 2x - 3y = 3 \\ -4x + 6y = -6 \end{cases}$

$m = -\frac{2}{-3} = \frac{2}{3}$

$m = \frac{4}{6} = \frac{2}{3}$

Infinite Solutions

Example 5 Solve each system by Elimination

$\begin{cases} 2(x - 2y = 3) \\ -2x + 4y = 1 \end{cases}$

$+\ 2x - 4y = 6$

$0 = 7$ false

No Solutions

$-2\begin{cases} (2x - y = 1) \\ 4x - 2y = 2 \end{cases}$

$-4x + 2y = -2$

$0 = 0$

True

Infinite Solutions

$4(40 \cdot 2)$

$160 \cdot 8$

$(16, 14)$

$\begin{cases} 100\ (0.02x - 0.05y = -0.38) \\ 100\ (0.03x + 0.04y = 1.04) \end{cases}$

$4(2x - 5y = -38)$

$5(3x + 4y = 104)$

$+\ 8x - 20y = -152$

$15x + 20y = 520$

$\frac{23x}{23} = \frac{368}{23}$

$x = 16$

$2(16) - 5y = -38$

$32 - 5y = -38$

$-32 \qquad -32$

$\frac{-5y}{-5} = \frac{-70}{-5}$

College Algebra Part of 6-3 and story problems Date: 12/10/13

Example 6 BREAK EVEN ANALYSIS
A shoe company invests $300,000 in equipment to produce a new line of athletic footwear. Each pair of shoes cost $5 to produce and is sold for $60. How many pairs of shoes must be sold before the business breaks even?

Output depend on the Input

x = Shoes

y = $

Cost $300{,}000 + 5x = y$

Profit $60x = y$

Cost = Profit

$$300{,}000 + 5x = 60x$$
$$-5x \quad -5x$$
$$\frac{300{,}000}{55} = \frac{55x}{55}$$
$$5455 = x$$

Sell 5455 shoes to break even

$$\begin{cases} x - 2y + 3z = 9 \\ y + 3z = 5 \\ z = 2 \end{cases}$$

$(1, -1, 2)$

$$x - 2y + 6 = 9$$
$$-6 \quad -6$$
$$x - 2y = 3$$

$$y + 6 = 5$$
$$-6 \quad -6$$
$$y = -1$$

$$x - 2(-1) = 3$$
$$x + 2 = 3$$
$$-2 \quad -2$$
$$x = 1$$

Example 8 An airplane flying into a head wind travels the 2000 mile flying distance between Chicopee, Massachusetts and Salt Lake City, Utah in 4 hours and 24 minutes. On the return flight, the same distance is traveled in 4 hours. Find the airspeed of the plane and the speed of the wind, assuming that both remain constant.

r_1 = Speed plane

r_2 = Wind Speed

$4\frac{24}{60}$ $\quad 4\frac{2}{5} = \frac{22}{5}$

$(r_1 - r_2) \rightarrow$ head Wind

$(r_1 + r_2) \rightarrow$ tail Wind

$$D = rt$$
$$2000 = (r_1 - r_2)\left(4 + \frac{24}{60}\right)$$
$$\frac{2000}{4} = \frac{(r_1 + r_2)(4)}{4}$$

$$\frac{5}{22} \cdot 2000 = (r_1 - r_2)\frac{22}{5} \cdot \frac{5}{22}$$

$$500 = r_1 + r_2$$
$$\frac{5000}{11} = r_1 - r_2$$

$$\frac{\overset{1000}{\cancel{2000}}}{1} \cdot \frac{5}{\underset{11}{\cancel{22}}}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 500 \\ \frac{5000}{11} \end{bmatrix}$$

$$x + y = 500$$
$$x - y = \frac{5000}{11}$$

$$"11(r_1 + r_2) = (500)11"$$
$$11r_1 - 11r_2 = 5000$$
$$+\ 11r_1 + 11r_2 = 5500$$
$$\frac{22r_1}{22} = \frac{10500}{22}$$
$$r_1 = 477.27 \text{ mph}$$

$r_2 = 22.72$ Mph

example 9
The demand and supply functions for a new type of personal digital assistant are

College Algebra Date: 1/2/13
7-2 7-3 MATRICES how to solve and inverses

Matrix: A rectangular array of real numbers Plural is matrices
Order: A matrix having m rows and n columns is said to be of order mxn
Square Matrices: m = n
Equal Matrices: Have the EXACT same ORDER and all elements are equal

$\begin{bmatrix}3 & -1\end{bmatrix}$ 1x2 $\begin{bmatrix}5 & 0 & -1\\ 6 & 7 & -3\\ 9 & -2 & 0\end{bmatrix}$ 3x3 $\begin{bmatrix}4\\ 2\\ -8\end{bmatrix}$ 3x1 $\begin{bmatrix}5\end{bmatrix}$ 1x1

Important Vocabulary Define each term or concept.

Scalars Real numbers used in operations with matrices.

Scalar multiple If $A = [a_{ij}]$ is an $m \times n$ matrix and c is a scalar, the scalar multiple of A by c is the $m \times n$ matrix given by $cA = [ca_{ij}]$.
Zero matrix A matrix consisting entirely of zeros.

Matrix multiplication If $A = [a_{ij}]$ is an $m \times n$ matrix and $B = [b_{ij}]$ is an $n \times p$ matrix, the product AB is an $m \times p$ matrix $AB = [c_{ij}]$ where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{in}b_{nj}$.
Identity matrix of order *n* The $n \times n$ matrix that consists of 1's on its main diagonal and 0's elsewhere.

$$5\underset{2\times3}{\begin{bmatrix}-1 & 3 & 2\\ 1 & 0 & 4\end{bmatrix}} = \underset{2\times3}{\begin{bmatrix}-5 & 15 & 10\\ 5 & 0 & 20\end{bmatrix}}$$

Example 1: Let $A = \begin{bmatrix}2 & 5\\ -3 & 1\end{bmatrix}$ and $B = \begin{bmatrix}-1 & 4\\ 2 & -5\end{bmatrix}$.

Find (a) $A + B$ and (b) $-2B$

A + B

$$\underset{2\times2}{\begin{bmatrix}2 & 5\\ -3 & 1\end{bmatrix}} + \underset{2\times2}{\begin{bmatrix}-1 & 4\\ 2 & -5\end{bmatrix}}$$

$$\underset{2\times2}{\begin{bmatrix}1 & 9\\ -1 & -4\end{bmatrix}}$$

$-2B$

$$-2\begin{bmatrix}-1 & 4\\ 2 & -5\end{bmatrix}$$

$$\begin{bmatrix}2 & -8\\ -4 & 10\end{bmatrix}$$

Let A, B, and C be $m \times n$ matrices and let c and d be scalars. Give an example of each of the following properties of matrix addition and scalar multiplication:

1) Commutative Property of Matrix Addition: $A + B = B + A$
2) Associative Property of Matrix Addition: $A + (B + C) = (A + B) + C$
3) Associative Property of Scalar Multiplication: $(cd)A = c(dA)$
4) Scalar Identity: $1A = A$
5) Distributive Property (two forms): $c(A + B) = cA + cB$
$(c + d)A = cA + dA$

If A is an $m \times n$ matrix and O is the $m \times n$ zero matrix consisting entirely of zeros, then $A + O =$ ___A___.

The additive identity for the set of all $m \times n$ matrices is the $m \times n$ matrix ___O (the zero matrix)___.

$$\underset{2\times3}{\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}}$$

III. Matrix Multiplication (Pages 542–544)

When multiplying an $m \times n$ matrix A by an $n \times p$ matrix B, to obtain the entry in the ith row and jth column of AB, . . . multiply the entries in the ith row of A by the corresponding entries in the jth column of B and then add the results.

A • B
$m \times \boxed{n \quad n} \times p$
$A \cdot B = m \times p$

Example 2: If A is a 3×5 matrix and B is a 6×3 matrix, find the order, if possible, of the product (a) AB, and (b) BA.

$$\begin{bmatrix}2 & 3\\ 4 & 8\end{bmatrix} \cdot \begin{bmatrix}1\\ 4\end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

$2 \times \boxed{2 \text{ same } 2} \times \boxed{1}$ 2×1
$m \times \boxed{n \cdot n} \times p = m \times p$

$A = 3 \times 5$ $B = 6 \times 3$
a) BA b) AB
$6 \times \boxed{3 \cdot 3} \times 5$ $3 \times 5 \cdot 6 \times 3$
6×5 Not Possibl

Example 3: Find the product AB, if

$$A = \begin{bmatrix}2 & -1 & 7\\ 0 & 6 & -3\end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix}0\\ -2\\ 3\end{bmatrix}$$

$$AB = 2 \times \overset{2\times3}{\boxed{3 \cdot 3}} \overset{3\times1}{\times 1} = 2 \times 1$$

$$\underset{2\times3}{\begin{bmatrix}2 & -1 & 7\\ 0 & 6 & -3\end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix}0\\ -2\\ 3\end{bmatrix}} = \underset{2\times1}{\begin{bmatrix}\dfrac{23}{2\cdot0+-1\cdot-2+7\cdot3}\\ \dfrac{-21}{0\cdot0+6\cdot-2+-3\cdot3}\end{bmatrix}}$$

$$\underset{2\times2}{\begin{bmatrix}3 & 4\\ -2 & 5\end{bmatrix}}\underset{2\times2}{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}} = \underset{2\times2}{\begin{bmatrix}\dfrac{3}{\cancel{3+0}} & \dfrac{4}{0+4}\\ \dfrac{-2}{-2+0} & \dfrac{5}{0+5}\end{bmatrix}}$$

Idenity Matrix

$$\begin{bmatrix}6 & 2 & 0\\ 3 & -1 & 2\\ 1 & 4 & 6\end{bmatrix}\begin{bmatrix}1\\ 2\\ -3\end{bmatrix} = \underset{3\times1}{\begin{bmatrix}\dfrac{10}{6+4+0}\\ \dfrac{-5}{3+-2+-6}\\ \dfrac{-9}{1+8+-18}\end{bmatrix}}$$

$3 \times \boxed{3 \quad 3} \times 1$

$$\underset{3\times2}{\begin{bmatrix}-2 & 1\\ 1 & -3\\ 1 & 4\end{bmatrix}}\underset{3\times4}{\begin{bmatrix}-2 & 3 & 1 & 4\\ 0 & 1 & -1 & 2\\ 2 & -1 & 0 & 1\end{bmatrix}} = \text{Not Possible}$$

College Algebra Lesson 7-3 inverse of a square matrix

Important Vocabulary Define each term or concept.

Square Matrix

Inverse of a matrix Let A be an $n \times n$ matrix and let I_n be the $n \times n$ identity matrix. If there exists a matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$ then A^{-1} is called the inverse of A.

$$\frac{3x}{3} = \frac{9}{3}$$

$$1X = 3$$

$$X = 3$$

identity matrices

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{2x2} \qquad \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3x3} \qquad \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{4x4}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse of a matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \begin{bmatrix} \frac{d}{\det} & \frac{-b}{\det} \\ \frac{-c}{\det} & \frac{a}{\det} \end{bmatrix}$$

$$\boxed{\det = ad - bc}$$

if det = 0 no Inverse

Example 1 show that B is an inverse of A

$$A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

Idenity Matrix

$$\underbrace{\begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}}_{2x2} \cdot \underbrace{\begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}}_{2x2} = \underbrace{\begin{bmatrix} \frac{1}{-1+2} & \frac{0}{2+-2} \\ \frac{0}{-1+1} & \frac{1}{2+-1} \end{bmatrix}}_{2x2}$$

$$\begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

$$A^{-1} \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 6 \\ -1 & 0 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} \frac{0}{-6} & \frac{-6}{-6} \\ \frac{1}{-6} & \frac{4}{-6} \end{bmatrix}$$

$$\det = 0 + -6$$

$$\det = -6 \qquad A^{-1} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{6} & \frac{-2}{3} \end{bmatrix}$$

College Algebra Solving systems of equations USING matrices

$$\begin{cases} x-4y+3z=5 \\ -x+3y-z=-3 \\ 2x+0y-4z=6 \end{cases}$$

$$\begin{cases} 0.02x-0.05y=-0.38 \\ 0.03x+0.04y=1.04 \end{cases}$$

$$\underset{A}{\begin{bmatrix} 1 & -4 & 3 \\ -1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underset{B}{\begin{bmatrix} 5 \\ -3 \\ 6 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\cdot B$$

$A^{-1}\cdot B$

$$\begin{cases} x-2y+3z=9 \\ -x+3y=-4 \\ 2x-5y+5z=17 \end{cases}$$

$$\underset{A}{\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underset{B}{\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\cdot B$$

APPLICATIONS!!

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Example 8 lesson 6.3

find a quadratic equation $y=ax^2+bx+c$

whose graph passes through the points (-1,3) (1,1) (2,6)

$3=a(-1)^2+b(-1)+c$

$1=a(1)^2+b(1)+c$

$6=a(2)^2+b(2)+c$

$1a-1b+c=3$

$1a+1b+c=1$

$4a+2b+c=6$

$$\underset{A}{\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \underset{B}{\begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = A^{-1}\cdot B$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

a) $y=2x^2+x+3$

b) $y=2x^2-x+4$

c) $y=2x^2-x-1$

d) $y=2x^2+0$

e) $\boxed{y=2x^2-x}$

Example 9 6.3***

An inheritance of \$12,000 was invested among three funds: a money-market that paid 5% annually, a mutual fund that paid 12% annually, and a municipal bond that paid 6% annually. The amount invested in the mutual funds was \$4000 more than the amount invested in the municipal bonds. The total interest earned during the first year was \$1120. How much was invested in EACH type of fund?

X = Money Market 5%

y = Mutal fund 12%

z = Municipal bond 6%

$$\begin{cases} x+y+z=12000 \\ y=4000+z \\ .05x+.12y+.06z=1120 \end{cases}$$

$x+y+z=12000$

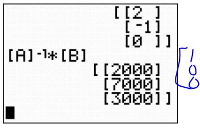
$.05x+.12y+.06z=1120$

$0x+y-z=4000$

$y=4000+z$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ .05 & .12 & .06 \\ 0 & 1 & -1 \end{bmatrix}} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underset{B}{\begin{bmatrix} 12000 \\ 1120 \\ 4000 \end{bmatrix}}$$

$$A^{-1}\cdot A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\cdot B$$



```
                [[2 ]
                 [-1]
                 [0 ]]
[A]-1*[B]
              [[2000]
               [7000]
               [3000]]
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 7000 \\ 3000 \end{bmatrix}$$

\$2000 invest in Money Market

\$7000 Mutal fund

\$3000 Municipal bond

College Algebra Lesson 6-5 Systems of inequalities

Important Vocabulary Define each term or concept.

Optimization A process in which the minimum or maximum of a quantity is sought.

Linear programming An optimization strategy.

Objective function Gives the quantity that is to be maximized or minimized.

Constraints A system of linear inequalities that determines the region of feasible solutions.

Feasible solutions All points in a region satisfying each of the constraints.

I. Linear Programming: A Graphical Approach

(Pages 501–504)

If a linear programming problem has a solution, it must occur . . .

at a vertex of the set of feasible solutions.

If there is more than one solution to a linear programming problem, at least one of them . . . must occur at a vertex.

In either case, the value of the objective function is unique.

List the steps for solving a linear programming problem:

1. Sketch the region corresponding to the system of constraints. The points inside or on the boundary of the region are the feasible solutions.
2. Find the vertices of the region.
3. Test the objective function at each of the vertices and select the values of the variables that optimize the objective function. For a bounded region both a minimum and a maximum value will exist. For an unbounded region, if an optimal solution exists, it will occur at a vertex.

Three equations for lines

Slope intercept form

$y = mx + b$

$m = slope$

$b = y-\text{int}\,(0,b)$

Standard form

$Ax + By = C$

$\rightarrow m = \frac{-A}{B}$

cover up method

$3x-4y=12$

$(4,0)$ $(0,-3)$

point slope form

PINK BLUE GREEN

$y - y_1 = m(x - x_1)$

$m = slope$

(x_1, y_1) $y-4=\frac{1}{2}(x+3)$

$(-3,4)$

$m=\frac{-3}{-4}$

$m=\frac{1}{2}$

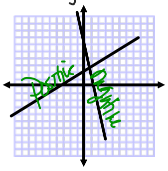
There are 3 Types of lines

OBLIQUE

$y=mx+b$

$Ax+By=C$

has a slope

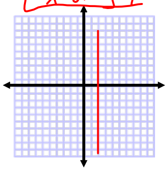


VERTICAL

$x=2$

x only

m=Undefined

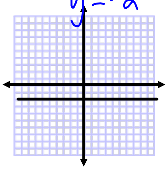


HORIZONTAL

y only

$y=-2$

$m=0$



Another way to graph

$y = ax^2 + c$ **or** $y = ax^2 + bx + c$

up down
wide or narrow

Parabola → Symmetric

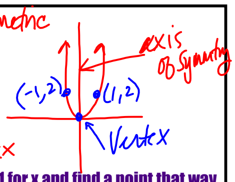
1. Find the vertex $\left(\frac{-b}{2a}, plug\,n\,chug\right)$

$\frac{-4}{2(2)}$

opens up

$y=2x^2+4x-6$ $\boxed{(-1,-8)}$ vertex

$y=2(-1)^2+4(-1)-6$



2. Find the y-intercept set x= 0 and solve for y . You can also plug in 1 for x and find a point that way

$(0,c)$

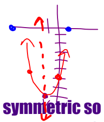
$2+-4-6$

$-2-6$

X	Y
2	8

$y=2x^2$ parent

x	y
2	8



Parabolas are symmetric so if you know the vertex and one point you can graph another point graph those 3 points and draw your parabola

4. Find the axis of symmetry and draw that in

$x = \frac{-b}{2a}$ $X=-1$

$y \geq \frac{1}{2}(x-3)^2+4$

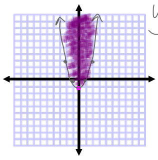
$(3,4)$

$y=\frac{1}{2}x^2$

x	y
-4	8
2	2
0	0
2	2
4	8

Example 1,2,3
Sketch the graph

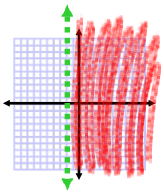
$y \ge x^2 - 1$ (0, -1)



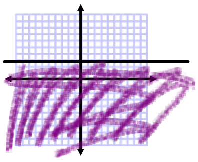
$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$x > -2$



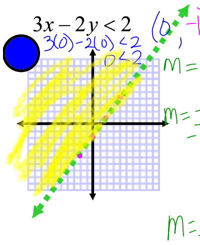
$y \le 3$



$3x - 2y < 2$ (0, -1)

$3(0) - 2(0) < 2$

$0 < 2$

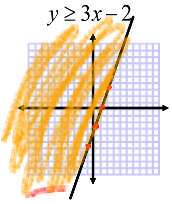


$m = -\frac{A}{B}$

$m = \frac{-3}{-2}$

$m = \frac{3}{2}$

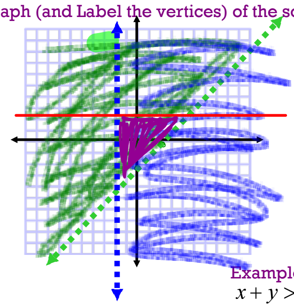
$y \ge 3x - 2$



Systems of inequalities
SOLUTION SET: The region that is common to every graph in the system

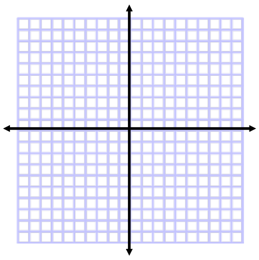
Example 4 sketch the graph (and Label the vertices) of the solution set

$x - y < 2$ ●
$x > -2$ ●
$y \le 3$ ●



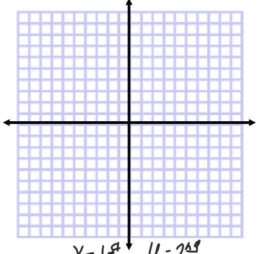
example 5

$x^2 - y \le 1$
$-x + y \le 1$



Example 6

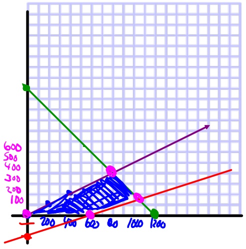
$x + y > 3$
$x + y < -1$



$x = 1^{st}$ $y = 2^{nd}$

$$\boxed{P = 1.50x + 2.00y}$$

● $x + y \le 1200$
● $y \le \frac{1}{2}x$ $2y \le x$
● $x \le 600 + 3y$

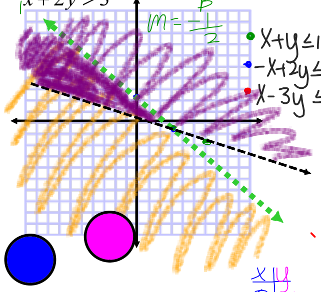


Example 7 an unbounded set

$x + y < 3$
$x + 2y > 3$

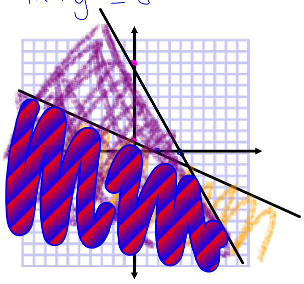
$m = -\frac{A}{B}$
$m = -\frac{1}{2}$

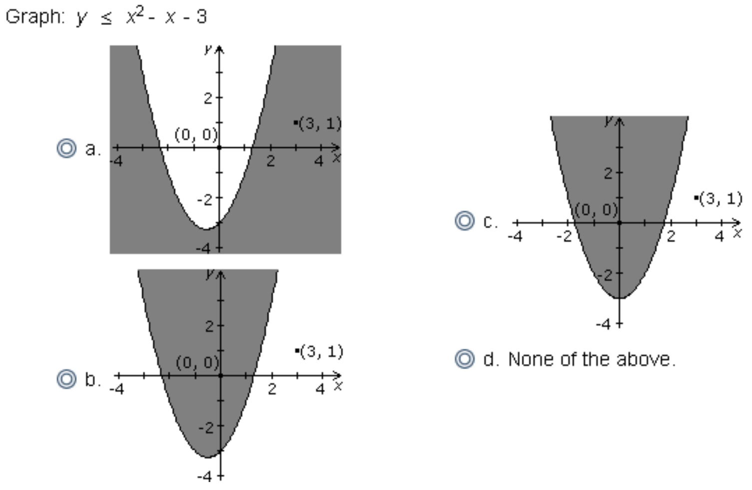
● $x + y \le 1200$
● $-x + 2y \le 0$
● $x - 3y \le 600$



$3x + 6y \le 6$
$2x + y \le 8$

x	y
0	1
2	0





Graph: $y \leq x^2 - x - 3$

○ a.

○ b.

○ c.

○ d. None of the above.

107. **Make a Decision** The table shows the earnings per share y for Wal-Mart from 1988 to 2003. *(Data Source: Wal-Mart Stores, Inc.)*

1988
$t=8$

Year	Earnings per share, y
8 1988	0.19
9 1989	0.24
10 1990	0.29
11 1991	0.35
12 1992	0.44
13 1993	0.51
14 1994	0.59
15 1995	0.60
16 1996	0.67
17 1997	0.78
18 1998	0.99
19 1999	1.28
20 2000	1.40
21 2001	1.50
22 2002	1.81
23 2003	2.03

(a) Use a graphing utility to plot the data. Let t represent the year, with $t = 8$ corresponding to 1988. Describe the rend in the data.

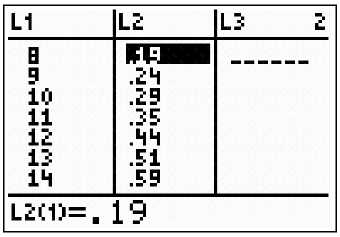
(b) Use the technique demonstrated in Exercises 67–70 in Section 6.3 to set up a system of equations for the data. Let t represent the year, with $t = 8$ corresponding to 1988.

(c) Use a graphing utility to graph the least squares regression parabola from part (b) and the original data in the same viewing window. How well does the model fit the data? Explain your reasoning.

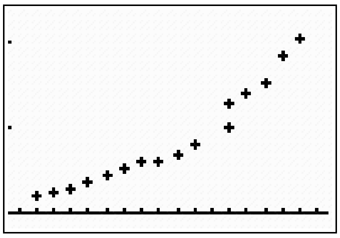
(d) Use the *regression* feature of a graphing utility to find a quadratic model for the data. How does the model given by the graphing utility compare with the model you found in part (b)?

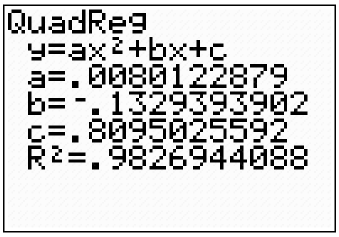
(e) Value Line projected that the earnings per share for Wal-Mart in 2004 and 2005 would be \$2.40 and \$2.75, respectively. Compare Value Line's projections for 2004 and 2005 with the earnings per share predicted by the model from part (b). Do Value Line's estimates agree with the model?

(f) Do you think a different type of model be a better fit for the data? If so, use the *regression* feature of a graphing utility to find another model for the data and predict the earnings per share for Wal-Mart in 2004 and 2005. Hoe do these predictions compare with those determined by Value Line?

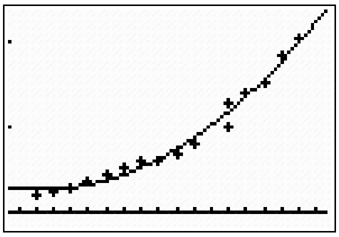


```
L1    |L2    |L3    2
 8    | .19  |------
 9    | .24  |
 10   | .29  |
 11   | .35  |
 12   | .44  |
 13   | .51  |
 14   | .59  |
L2(1)=.19
```





```
QuadReg
 y=ax²+bx+c
 a=.0080122879
 b=-.1329393902
 c=.8095025592
 R²=.9826944088
```



$y=at^2+bt+c$

$.19=a(8^2)+b\cdot 8+c$

$.60=a(15^2)+b\cdot 15+c$

$2.03=a(23)^2+b\cdot 23+c$

$64a+8b+c=.19$

$225a+15b+c=.6$

$529a+23b+c=2.03$

$$\begin{bmatrix}64&8&1\\225&15&1\\529&23&1\end{bmatrix}\cdot\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}.19\\.6\\2.03\end{bmatrix}$$

$$\boxed{b)\ y=.008t^2-.126t+.683}$$

Chapter 6 Project ▶ Fitting Models to Data

Many of the models in this book were created with a statistical method called *least squares regression analysis.* This procedure can be performed easily with a computer or graphing utility.

Example ▶ Fitting a Line to Data

The numbers of morning and evening newspapers published in the United States from 1993 through 2000 are shown in the table. Use the data to predict the numbers of morning and evening newspapers that will be published in 2005. In the table, $t = 3$ represents 1993. (Source: Editor and Publisher Company)

Year, t	3	4	5	6	7	8	9	10
Morning	623	635	656	686	705	721	736	766
Evening	954	935	891	846	816	781	760	727

Chapter Project Investigations

1. Use the models from the example to estimate the year in which the number of morning papers published was equal to the number of evening papers published.
2. The total numbers (in millions) of *all* morning newspapers *sold* each day in the United States from 1993 to 2000 are shown in the table at the left. Find a linear model that represents this data. Use your model to predict the number of morning papers that will be sold each day in 2005.
3. From 1993 through 2000, both the number of morning newspapers published and the total number of papers sold increased. Did the average circulation per morning paper (number sold ÷ number published) increase or decrease? Explain.

- * Solve Systems of 2 equations → Must do one by Substitution
- * Solve Systems of 3 equations
- * graph a Circle $(x-h)^2+(y-k)^2=r^2$ (h,k) Center radius = r
  $x^2+y^2=r^2$ $(0,0)$
- * graph a System of Inequalities (Don't forget all of the functions we have graphed)
- * factor a quadratic (build our house)
- * Review of log rules

Graphing Review

Lines → degree of 1

$Ax+By=C$ $m=-\frac{A}{B}$ Cover-up

$y=mx+b$

$x=4$ Vertical

$y=7$ Horizontal

Parabolas degree → 2

$y=ax^2+bx+c$

Vertex $\left(\frac{-b}{2a}, P;c\right)$

Parent function

$y=ax^2$

x	y
-2	
-1	
0	
1	
2	

axis of Symmetry

$x=\frac{-b}{2a}$

$y=a(x-h)^2+k$

Vertex form

(h,k) vertex

Parent function

$y=ax^2$

x	y
-2	
-1	
0	
1	
2	

Example Standard form

Graph $y \geq -\frac{1}{2}x^2+4x+5$

Vertex $\left(\frac{-b}{2a}, P;c\right)$

$\frac{-4}{2(-\frac{1}{2})}$ $(4, 13)$

$-\frac{1}{2}(4)^2+4(4)+5$

$-8+16+5$

$8+5$

13

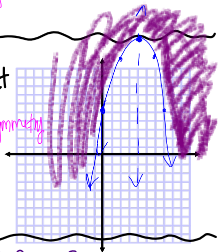
Vertex $(4,13)$

$y=-\frac{1}{2}x^2$ Parent

x	y
-4	-8
-2	-2
0	0
2	-2
4	-8

axis of Symmetry

$x=4$



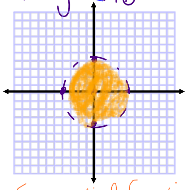
Circles

$x^2+y^2=r^2$

Center $(0,0)$

Radius = r

$x^2+y^2 \leq 16$

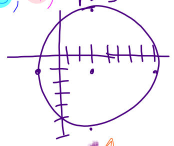


$(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y+1)^2=25$

$(x-3)^2+(y--1)^2=25$

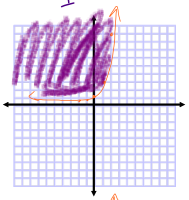
$(3,-1)$ $r=5$



Exponential functions

$y \geq 3^x$

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9



$(0,0)$

$y \geq 3^x$

$0 \geq 3^0$

$0 \geq 1$

no

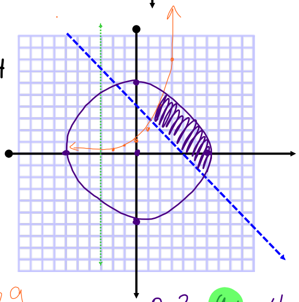
$x>-3$

$x+y>4$ $y>-x+4$

$y \leq 2^x$

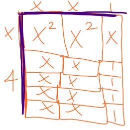
$x^2+y^2 \leq 36$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4



ac
b

$2x^2+9x+4=0$ $\frac{8}{8,1 \; 9}$



$(2x+1)(x+4)=0$

$x=-\frac{1}{2}$ $x=-4$

$2x^2+9x+4=0$

$\boxed{2x^2+8x}+1x+4=0$

$2x(x+4)+1(x+4)=0$

$(2x+1)(x+4)=0$

$\log_2 16=4$

$2^4=16$

$\log_2 \sqrt{16x}$

$\log_2\sqrt{16}+\log_2\sqrt{x}$

$2^{\square}=4$

$\log_2 4+\log_2\sqrt{x}$

$2+\log_2 x^{\frac{1}{2}}$

$2+\frac{1}{2}\cdot\log_2 x$

Attachments

- practice test solutions .pdf
- Chapter 1 MULT choice.pdf
- 6.4 Teacher notetaking.pdf
- 6-2 notetaking.pdf
- 6-2 teacher notetaking.pdf
- 6-4 notetaking.pdf
- 6-5 notetaking.pdf
- 6-6 notetaking.pdf
- 6.3 Notetaking.pdf
- 6.3 teacher notetaking.pdf