

## College Algebra

Date: 11/16/15

## 4-1 Exponential functions

$$y = b^x \text{ or } f(x) = b^x$$

$$b \neq 1$$

Example 1: the function  $f(x) = 42.2(1.56)^x$  models the average amount spent,  $f(x)$ , in dollars, at a shopping mall after  $x$  hours. What is the average amount spent, to the nearest dollar, after 4 hours?

$$f(x) = 42.2(1.56)^4$$

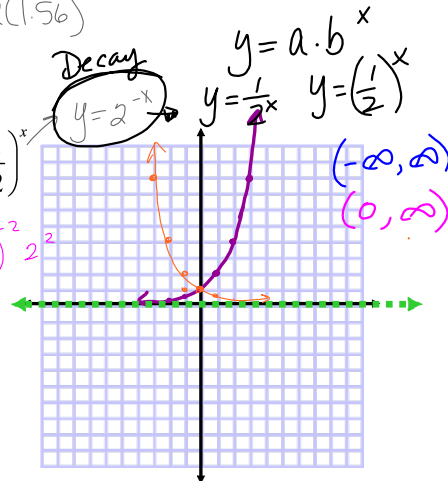
Example 2: Graph the function

Growth  
 $y = 2^x$

| x  | y             |
|----|---------------|
| -2 | $\frac{1}{4}$ |
| -1 | $\frac{1}{2}$ |
| 0  | 1             |
| 1  | 2             |
| 2  | 4             |

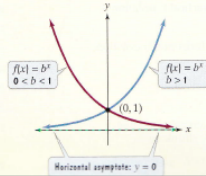
Decay  
 $f(x) = \left(\frac{1}{2}\right)^x$

| x  | y             |
|----|---------------|
| -2 | 4             |
| -1 | 2             |
| 0  | 1             |
| 1  | $\frac{1}{2}$ |
| 2  | $\frac{1}{4}$ |

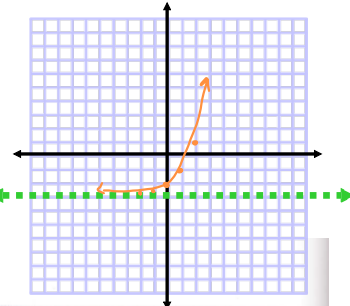


Characteristics of Exponential Functions of the Form  $f(x) = b^x$

- The domain of  $f(x) = b^x$  consists of all real numbers:  $(-\infty, \infty)$ . The range of  $f(x) = b^x$  consists of all positive real numbers:  $(0, \infty)$ .
- The graphs of all exponential functions of the form  $f(x) = b^x$  pass through the point  $(0, 1)$  because  $f(0) = b^0 = 1$  ( $b \neq 0$ ). The y-intercept is 1. There is no x-intercept.
- If  $b > 1$ ,  $f(x) = b^x$  has a graph that goes up to the right and is an increasing function. The greater the value of  $b$ , the steeper the increase.
- If  $0 < b < 1$ ,  $f(x) = b^x$  has a graph that goes down to the right and is a decreasing function. The smaller the value of  $b$ , the steeper the decrease.
- $f(x) = b^x$  is one-to-one and has an inverse that is a function.
- The graph of  $f(x) = b^x$  approaches, but does not touch, the x-axis. The x-axis, or  $y = 0$ , is a horizontal asymptote.



$y = 2^x$  down 3  
 $y = 2^{x-3}$   
 $2^{-2} = \frac{1}{2^2}$   
 $2^{-1} = \frac{1}{2^1}$   
 $y = 3^{x+2}$  left 2



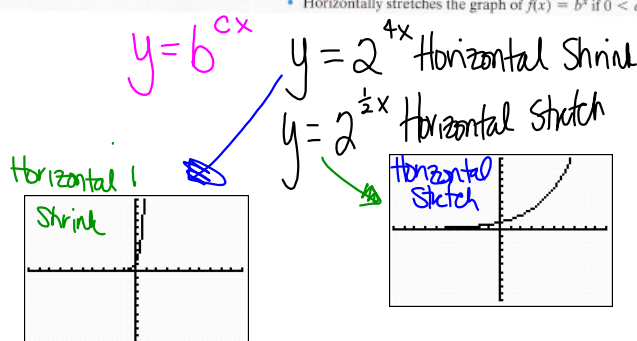
### Transformations of Exponential Functions

The graphs of exponential functions can be translated vertically or horizontally, reflected, stretched, or shrunk. These transformations are summarized in Table 4.1.

Table 4.1 Transformations Involving Exponential Functions

In each case,  $c$  represents a positive real number.

| Transformation                     | Equation                             | Description                                                                                                                                                                                                                                 |
|------------------------------------|--------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Vertical translation               | $g(x) = b^x + c$<br>$g(x) = b^x - c$ | <ul style="list-style-type: none"> <li>Shifts the graph of <math>f(x) = b^x</math> upward <math>c</math> units.</li> <li>Shifts the graph of <math>f(x) = b^x</math> downward <math>c</math> units.</li> </ul>                              |
| Horizontal translation             | $g(x) = b^{x+c}$<br>$g(x) = b^{x-c}$ | <ul style="list-style-type: none"> <li>Shifts the graph of <math>f(x) = b^x</math> to the left <math>c</math> units.</li> <li>Shifts the graph of <math>f(x) = b^x</math> to the right <math>c</math> units.</li> </ul>                     |
| Reflection                         | $g(x) = -b^x$<br>$g(x) = b^{-x}$     | <ul style="list-style-type: none"> <li>Reflects the graph of <math>f(x) = b^x</math> about the x-axis.</li> <li>Reflects the graph of <math>f(x) = b^x</math> about the y-axis.</li> </ul>                                                  |
| Vertical stretching or shrinking   | $g(x) = cb^x$                        | <ul style="list-style-type: none"> <li>Vertically stretches the graph of <math>f(x) = b^x</math> if <math>c &gt; 1</math>.</li> <li>Vertically shrinks the graph of <math>f(x) = b^x</math> if <math>0 &lt; c &lt; 1</math>.</li> </ul>     |
| Horizontal stretching or shrinking | $g(x) = b^{cx}$                      | <ul style="list-style-type: none"> <li>Horizontally shrinks the graph of <math>f(x) = b^x</math> if <math>c &gt; 1</math>.</li> <li>Horizontally stretches the graph of <math>f(x) = b^x</math> if <math>0 &lt; c &lt; 1</math>.</li> </ul> |



Natural Base  $e$  $e$  is an irrational number

Table 4.2

| $n$           | $\left(1 + \frac{1}{n}\right)^n$ |
|---------------|----------------------------------|
| 1             | 2                                |
| 2             | 2.25                             |
| 5             | 2.48832                          |
| 10            | 2.59374246                       |
| 100           | 2.704813829                      |
| 1000          | 2.716923932                      |
| 10,000        | 2.718145927                      |
| 100,000       | 2.718268237                      |
| 1,000,000     | 2.718280469                      |
| 1,000,000,000 | 2.718281827                      |

As  $n \rightarrow \infty$ ,  $\left(1 + \frac{1}{n}\right)^n \rightarrow e$ .

In calculus, this is expressed as  
 $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ .

## EXAMPLE 6 Gray Wolf Population

Insatiable killer. That's the reputation the gray wolf acquired in the United States in the nineteenth and early twentieth centuries. Although the label was undeserved, an estimated two million wolves were shot, trapped, or poisoned. By 1960, the population was reduced to 800 wolves. **Figure 4.6** shows the rebounding population in two recovery areas after the gray wolf was declared an endangered species and received federal protection.

Gray Wolf Population in Two Recovery Areas for Selected Years

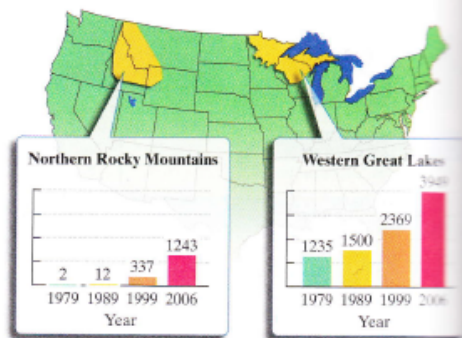


FIGURE 4.6

Source: U.S. Fish and Wildlife Service

The exponential function

$$f(x) = 1.26e^{0.247x}$$

models the gray wolf population of the Northern Rocky Mountains,  $f(x)$ ,  $x$  years after 1978. If the wolf is not removed from the endangered species list and trends shown in **Figure 4.6** continue, project its population in the recovery area in 2010.

$$f(x) = 1.26e^{0.247(32)}$$

Compound interest: is the interest computed on your original investment as well as on any accumulated interest.

6% Interest Rate

## Formulas for Compound Interest

After  $t$  years, the balance,  $A$ , in an account with principal  $P$  and annual interest rate  $r$  (in decimal form) is given by the following formulas:

1. For  $n$  compounding periods per year:  $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. For continuous compounding:  $A = Pe^{rt}$

$$\begin{matrix} n \cdot t \\ 12 \cdot 5 \\ 60 \end{matrix}$$

Table 4.4 Interest Plans

| Name                   | Number of Compounding Periods per Year | Length of Each Compounding Period |
|------------------------|----------------------------------------|-----------------------------------|
| Semiannual Compounding | $n = 2$                                | 6 months                          |
| Quarterly Compounding  | $n = 4$                                | 3 months                          |
| Monthly Compounding    | $n = 12$                               | 1 month                           |

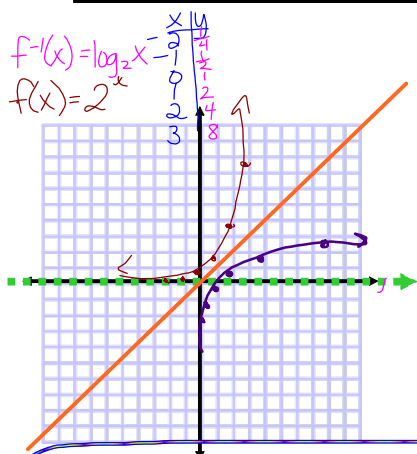
## EXAMPLE 7 Choosing between Investments

You decide to invest \$8000 for 6 years and you have a choice between two accounts. The first pays 7% per year, compounded monthly. The second pays 6.85% per year, compounded continuously. Which is the better investment?

$$\begin{aligned} A &= Pe^{rt} \\ A &= 500e^{0.25 \cdot 5} \end{aligned}$$

## 4-2 Log Functions

|                             |                               |                       |
|-----------------------------|-------------------------------|-----------------------|
| $y = \log_b x$<br>$b^y = x$ | $2^4 = 16$<br>$\log_2 16 = 4$ | Exponents<br>Log form |
|-----------------------------|-------------------------------|-----------------------|



Inverse

- Reflections over  $y=x$
- Switched  $x$  &  $y$  Solved for  $y$
- Domain  $f(x)$  = Range  $f^{-1}(x)$
- Range  $f(x)$  = Domain  $f^{-1}(x)$
- $f(f^{-1}(x)) = x$   $f^{-1}(f(x)) = x$

$$x = 2^y$$

$$\log_2 x = y$$

| x   | y  |
|-----|----|
| 1/4 | -2 |
| 1/2 | -1 |
| 1   | 0  |
| 2   | 1  |
| 4   | 2  |
| 8   | 3  |

|                           |                             |
|---------------------------|-----------------------------|
| $2^4 = 16$<br>Exponential | $\log_2 16 = 4$<br>Log form |
|---------------------------|-----------------------------|

Example: Write in exponential form

$$2 = \log_5 x$$

$$5^2 = x$$

$$x = 25$$

$$3 = \log_b 64$$

$$b^3 = 64$$

$$b = 4$$

|                   |
|-------------------|
| Write in log form |
| $12^2 = x$        |
| $\log_{12} x = 2$ |

$$4^{-3} = \frac{1}{64} \log_4 \frac{1}{64} =$$

Evaluate the log

$$\log_2 16 = \boxed{4}$$

$$2^{\square} = 16$$

$$\log_7 \frac{1}{49} = -2$$

$$7^{\square} = \frac{1}{49}$$

$$7^{\square} = \frac{1}{7^2}$$

$$\log_x 9 = 2$$

$$x^2 = 9$$

$$\log_3 9 = 2$$

$$\log_{10} 100 = 2$$

$$10^{\square} = 100$$

$$\log_{36} 6 = \frac{1}{2}$$

$$36^{\square} = 6$$

$$\log_3 3^{\frac{1}{2}} = \boxed{\frac{1}{2}}$$

$$3^{\square} = 3^{\frac{1}{2}}$$

$$\log_3 \sqrt[3]{3} = \frac{1}{3}$$

$$3^{\square} = \sqrt[3]{3}$$

$$3^{\square} = 3^{\frac{1}{3}}$$

Same problems

$$b^0 =$$

Properties of logs

|                                     |                                             |
|-------------------------------------|---------------------------------------------|
| $\log_b b = 1$<br>$b^{\square} = b$ | $\log_5 5 = \boxed{1}$<br>$5^{\square} = 5$ |
|-------------------------------------|---------------------------------------------|

|                                             |
|---------------------------------------------|
| $\log_b 1 = \boxed{0}$<br>$b^{\square} = 1$ |
|---------------------------------------------|

|                                                 |                    |
|-------------------------------------------------|--------------------|
| $\log_b b^x = \boxed{x}$<br>$b^{\square} = b^x$ | $b^{\log_b x} = x$ |
|-------------------------------------------------|--------------------|

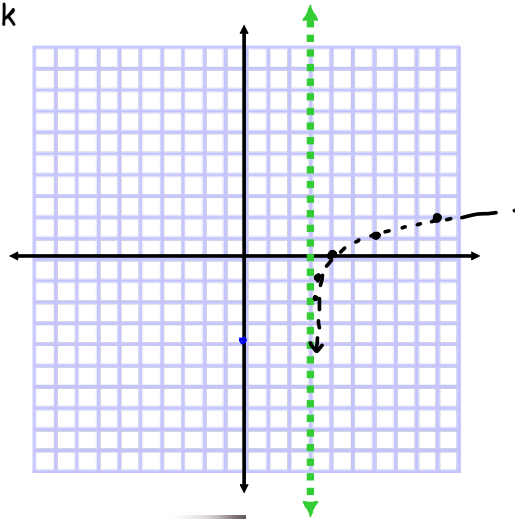
# Transformations of logs. h and k

## Find the domain

$y = \log_3(x-3)$  Right 3 (3, ∞)  
 $y = \log_3 x$   
 $3^y = x$   

|     |    |
|-----|----|
| x   | y  |
| 1/3 | -1 |
| 1   | 0  |
| 3   | 1  |
| 9   | 2  |

down 4  
 $y = \log_3 x - 4$



### Characteristics of Logarithmic Functions of the Form $f(x) = \log_b x$

1. The domain of  $f(x) = \log_b x$  consists of all positive real numbers:  $(0, \infty)$ . The range of  $f(x) = \log_b x$  consists of all real numbers:  $(-\infty, \infty)$ .
2. The graphs of all logarithmic functions of the form  $f(x) = \log_b x$  pass through the point  $(1, 0)$  because  $f(1) = \log_b 1 = 0$ . The  $x$ -intercept is 1. There is no  $y$ -intercept.
3. If  $b > 1$ ,  $f(x) = \log_b x$  has a graph that goes up to the right and is an increasing function.
4. If  $0 < b < 1$ ,  $f(x) = \log_b x$  has a graph that goes down to the right and is a decreasing function.
5. The graph of  $f(x) = \log_b x$  approaches, but does not touch, the  $y$ -axis. The  $y$ -axis, or  $x = 0$ , is a vertical asymptote.

| Transformation                     | Equation               | Description                                                                                                                             |
|------------------------------------|------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| Vertical translation               | $g(x) = \log_b x + c$  | Shifts the graph of $f(x) = \log_b x$ upward $c$ units.                                                                                 |
|                                    | $g(x) = \log_b x - c$  | Shifts the graph of $f(x) = \log_b x$ downward $c$ units.                                                                               |
| Horizontal translation             | $g(x) = \log_b(x + c)$ | Shifts the graph of $f(x) = \log_b x$ to the left $c$ units. Vertical asymptote: $x = -c$ .                                             |
|                                    | $g(x) = \log_b(x - c)$ | Shifts the graph of $f(x) = \log_b x$ to the right $c$ units. Vertical asymptote: $x = c$ .                                             |
| Reflection                         | $g(x) = -\log_b x$     | Reflects the graph of $f(x) = \log_b x$ about the $x$ -axis.                                                                            |
|                                    | $g(x) = \log_b(-x)$    | Reflects the graph of $f(x) = \log_b x$ about the $y$ -axis.                                                                            |
| Vertical stretching or shrinking   | $g(x) = c \log_b x$    | Vertically stretches the graph of $f(x) = \log_b x$ if $c > 1$ . Vertically shrinks the graph of $f(x) = \log_b x$ if $0 < c < 1$ .     |
| Horizontal stretching or shrinking | $g(x) = \log_b(cx)$    | Horizontally shrinks the graph of $f(x) = \log_b x$ if $c > 1$ . Horizontally stretches the graph of $f(x) = \log_b x$ if $0 < c < 1$ . |

$2^4 = 16$   
 $\log_2 16 = 4$

$\log 100 = 2$   
 $10^2 = 100$   
 $2^4 = 16$   
 $\log_2 16 = 4$

### Properties of Common Logarithms

**General Properties**

1.  $\log_b 1 = 0$
2.  $\log_b b = 1$
3.  $\log_b b^x = x$
4.  $b^{\log_b x} = x$

**Inverse properties**

$b^{\log_b x} = x$

**Common Logarithms**

1.  $\log 1 = 0$
2.  $\log 10 = 1$
3.  $\log 10^x = x$
4.  $10^{\log x} = x$

**Natural logs**

$\ln 1 = 0$

$\ln e = 1$

$\ln e^x = x$

$e^{\ln x} = x$

$\log_2 4 = \log_2 2^2 = 2$

$\log_{10} 1 = 0$

$\log_e e = 1$

**Common log**  
 $\log_{10} 1 = \log 1$

**Natural logs**  
 $\log_e 1 = \ln 1$

### EXAMPLE 8 Modeling Height of Children

The percentage of adult height attained by a boy who is  $x$  years old can be modeled by

$$f(x) = 29 + 48.8 \log(x + 1),$$

where  $x$  represents the boy's age (from 5 to 15) and  $f(x)$  represents the percentage of his adult height. Approximately what percentage of his adult height has he attained at age eight?

## Lesson 4-3 Properties of Logs

$$\begin{aligned}
 \log_b(MN) &= \log_b M + \log_b N && \text{Product Rule} \\
 \log_4(16 \cdot 4) &= \log_4 16 + \log_4 4 \\
 \log_b\left(\frac{M}{N}\right) &= \log_b M - \log_b N && \text{Quotient Rule} \\
 \log_4 \frac{16}{4} &= \log_4 16 - \log_4 4 \\
 \log_b M^p &= p \cdot \log_b M && \text{Power Rule} \\
 \log_4 16^2 &= 2 \cdot \log_4 16 \\
 \log_b M &= \frac{\log_a M}{\log_a b} && \text{Change-of-base Property} \\
 \log_2 32 &= \frac{\log 32}{\log 2}
 \end{aligned}$$

$$\begin{aligned}
 a^m \cdot a^n &= a^{m+n} && x^4 \cdot x^2 = x^6 \\
 \frac{a^m}{a^n} &= a^{m-n} && \frac{x^7}{x^3} = x^4 \\
 (a^m)^n &= a^{m \cdot n} && (x^4)^2 = x^8
 \end{aligned}$$

Common Logarithms

$$\begin{aligned}
 \log_b M &= \frac{\log_{10} M}{\log_{10} b} \\
 \log_2 32 &= \frac{\log 32}{\log 2} \\
 \log_b M &= \frac{\log M}{\log b}
 \end{aligned}$$

Natural Logarithms

$$\begin{aligned}
 \log_b M &= \frac{\log_e M}{\log_e b} \\
 \log_2 32 &= \frac{\ln 32}{\ln 2} \\
 \log_b M &= \frac{\ln M}{\ln b}
 \end{aligned}$$

Example 1: Explain using Properties Example 2:

$$\begin{aligned}
 \log_4(7 \cdot 5) &= \log_4 7 + \log_4 5 \\
 \log_4 7 + \log_4 5
 \end{aligned}$$

$$\log(10x)$$

$$\log_7\left(\frac{19}{x}\right)$$

$$\ln\left(\frac{e^3}{7}\right)$$

$$\ln e^3 - \ln 7$$

Ex 3:

$$\begin{aligned}
 \log_5 7^4 &= 4 \cdot \log_5 7
 \end{aligned}$$

$$\ln \sqrt{x}$$

$$\log(4x)^5$$

$$\begin{aligned}
 3 \cdot \ln e - \ln 7 \\
 3 - \ln 7
 \end{aligned}$$

Ex 4:

$$\log_b(x^2 y^{\frac{1}{2}})$$

$$\log_b\left(\frac{\sqrt[3]{x}}{36y^4}\right)^{x^2}$$

Ex 5: Write as a Single Log

$$\begin{aligned}
 \log_4 2 + \log_4 32 \\
 \log_4(2 \cdot 32) \\
 \log_4(64) = 3
 \end{aligned}$$

$$\log\left(\frac{4x+3}{x}\right) - \log x$$

$$\begin{aligned}
 \log 25 + \log 4 \\
 \log(25 \cdot 4) \\
 \log 100 = 2
 \end{aligned}$$

$$\begin{aligned}
 \log(7x+6) - \log x \\
 \log\left(\frac{7x+6}{x}\right)
 \end{aligned}$$

Ex 6:

$$\begin{aligned}
 \frac{1}{2} \log x + 4 \log(x-1) \\
 \log x^{\frac{1}{2}} + \log(x-1)^4 \\
 \log x^{\frac{1}{2} \cdot (x-1)^4} \\
 \log(\sqrt{x} \cdot (x-1)^4)
 \end{aligned}$$

$$\begin{aligned}
 3 \ln(x+7) - \ln x \\
 \ln(x+7)^3 - \ln x \\
 \ln\left(\frac{(x+7)^3}{x}\right)
 \end{aligned}$$

$$\begin{aligned}
 4 \log_b x - 2 \log_b 6 - \frac{1}{2} \log_b y \\
 \log_b x^4 - \log_b 6^2 - \log_b y^{\frac{1}{2}} \\
 \log_b \frac{x^4}{36 \sqrt{y}}
 \end{aligned}$$

$$\log_b M = \frac{\log_a M}{\log_a b}$$

Change-of-base Property

Use common logs to evaluate

$$\log_5 140 \quad 5^x = 140 \quad \log_7 2506$$

$$x = \frac{\log 140}{\log 5} \quad \log 5^x = \log 140$$

$$\frac{x \cdot \log 5 = \log 140}{\log 5}$$

Use Natural logs to evaluate

$$\log_5 140 = \frac{\ln 140}{\ln 5}$$

$$\log_7 2506$$

$$5^x = 140$$

How Do you graph  $y = \log_2 x$  on Calculator?

## 4-4 Exponential and log functions and solving for x

$$(a^m)^n = a^{m \cdot n}$$

Exponential equations Example 1  $b^M = b^N$ , then  $M = N$ 

$$2^{3x-8} = 16$$

$$2^{3x-8} = 2^4$$

$$\begin{array}{r} 3x-8 = 4 \\ +8 \quad +8 \\ \hline 3x = 12 \\ \div 3 \quad \div 3 \\ \hline x = 4 \end{array}$$

$$27^{x+3} = 9^{x-1}$$

$$\begin{array}{r} 3^{3(x+3)} = 3^{2(x-1)} \\ 3^{3x+9} = 3^{2x-2} \\ 3x+9 = 2x-2 \\ -2x \quad -2x \\ \hline 1x+9 = -2 \\ -9 \quad -9 \\ \hline x = -11 \end{array}$$

$$5^{3x-6} = 125$$

$$\begin{array}{r} 5^{3x-6} = 5^3 \\ 3x-6 = 3 \\ +6 \quad +6 \\ \hline 3x = 9 \\ \div 3 \quad \div 3 \\ \hline x = 3 \end{array}$$

$$8^{x+1} = 4^{x-3}$$

Example 2 and 3

$$4^x = 15$$

$$\log 4^x = \log 15$$

$$\frac{x \cdot \log 4}{\log 4} = \frac{\log 15}{\log 4}$$

$$x = \frac{\log 15}{\log 4}$$

Whenever the Variable is the exponent TAKE THE LOG or LN of both Sides.

$$10^x = 120000$$

$$\log 10^x = \log 120000$$

$$\frac{x \cdot \log 10}{\log 10} = \frac{\log 120000}{\log 10}$$

$$x = \log 120000$$

$$40e^{0.6x} = 240$$

$$e^{0.6x} = 6$$

$$\ln e^{0.6x} = \ln 6$$

$$\frac{0.6x \cdot \ln e}{\ln e} = \frac{\ln 6}{\ln e}$$

$$x = 2.99$$

$$7e^{2x} - 5 = 58$$

$$\begin{array}{r} 7e^{2x} - 5 = 58 \\ +5 \quad +5 \\ \hline 7e^{2x} = 63 \\ \div 7 \quad \div 7 \\ \hline e^{2x} = 9 \end{array}$$

$$\ln e^{2x} = \ln 9$$

$$\begin{array}{r} 2x \cdot \ln e = \ln 9 \\ \frac{2x}{2} = \frac{\ln 9}{2} \quad x = \frac{\ln 9}{2} \\ (x^m)^n = x^{m \cdot n} \end{array}$$

Example 4 and 5

$$5^{x-2} = 4^{2x+3}$$

$$e^2 - 4e + 3 = 0$$

$$e^{2x} - 4e^x + 3 = 0$$

$$m = e^x$$

$$(e^x)^2 - 4e^x + 3 = 0$$

$$m^2 - 4m + 3 = 0$$

$$(m-3)(m-1) = 0$$

$$m = 3 \quad m = 1$$

$$e^x = 3 \quad e^x = 1$$

$$\ln e^x = \ln 3$$

$$x \cdot \ln e = \ln 3$$

$$\frac{x \cdot \ln e}{\ln e} = \frac{\ln 3}{\ln e}$$

$$x = \ln 3$$

$$\ln e^x = \ln 1$$

$$\frac{x \cdot \ln e}{\ln e} = \frac{\ln 1}{\ln e}$$

$$x = 0$$

$$\begin{array}{r} 3 \overline{) 17} -4 \\ m^2 - 4m + 3 = 0 \\ (m-3)(m-1) = 0 \\ m = 3 \quad m = 1 \end{array}$$

4-4 Log equations Example 6  $2^4=16 \quad \log_2 16=4$ 

$$\log_4(x+3)=2$$

$$4^2 = x+3$$

$$\begin{array}{r} 16 = x+3 \\ -3 \quad -3 \\ \hline \end{array}$$

$$13 = x$$

$$\log_2(x-4)=3$$

$$2^3 = x-4$$

$$4 \ln(3x) = 8$$

$$\ln(3x)^4 = 8$$

$$(e^{\ln(3x)^4})^{\frac{1}{4}} = (e^8)^{\frac{1}{4}}$$

$$\frac{e^2}{3} = \frac{3x}{3}$$

$$\frac{e^2}{3} = x$$

## Example 7

$$\log_2 16 = 4 \quad 2^4 = 16 \quad 2^D = 8 \quad 2^D = 1$$

$$\log_2 x + \log_2(x-7) = 3 \quad \log_2 8 + \log_2 1 = 3 \quad \log x + \log(x-3) = 1$$

$$\log_2(x(x-7)) = 3$$

$$3+0=3 \quad \log_2 1 + \log_2(-9) = 3$$

$$\log(x(x-3)) = 1$$

$$\log_2 x^2 - 7x = 3 \quad 2^3 = -1$$

$$\log x^2 - 3x = 1$$

$$2^3 = x^2 - 7x$$

$$10^1 = x^2 - 3x$$

$$\begin{array}{r} -8 \\ -8 \end{array} \quad \begin{array}{r} 0 = x^2 - 7x - 8 \\ 0 = (x-8)(x+1) \end{array}$$

$$x=8 \quad x=-1 \text{ (extraneous)}$$

## ONE-TO-ONE Property

## Using the One-to-One Property of Logarithms to Solve Logarithmic Equations

- Express the equation in the form  $\log_b M = \log_b N$ . This form involves a single logarithm whose coefficient is 1 on each side of the equation.
- Use the one-to-one property to rewrite the equation without logarithms. If  $\log_b M = \log_b N$ , then  $M = N$ .
- Solve for the variable.
- Check proposed solutions in the original equation. Include in the solution set only values for which  $M > 0$  and  $N > 0$ .

$$M = N$$

$$\log_2 16 = \log_2 x$$

$$16 = x$$

## EXAMPLE 8 Solving a Logarithmic Equation

$$\text{Solve: } \ln(x+2) - \ln(4x+3) = \ln\left(\frac{1}{x}\right)$$

$$\ln\left(\frac{x+2}{4x+3}\right) = \ln\left(\frac{1}{x}\right)$$

$$\frac{x+2}{4x+3} = \frac{1}{x}$$

$$4x+3 = x(x+2)$$

$$4x+3 = x^2+2x$$

$$\begin{array}{r} -4x-3 \quad -4x-3 \\ \hline \end{array}$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$$x=3 \quad x=-1 \text{ (extraneous)}$$

$$\ln 5 - \ln 15 = \ln \frac{1}{3} \quad \ln 1 - \ln(-1)$$

$$\ln \frac{5}{15} = \ln \frac{1}{3}$$

$$e^0 = -1$$

Cannot take the  $\uparrow$  log of a negative  $\ln(-1)$  Not possible

But you cannot take the  $\ln$  of  $-1$

**EXAMPLE 9** Alcohol and Risk of a Car Accident

Medical research indicates that the risk of having a car accident increases exponentially as the concentration of alcohol in the blood increases. The risk is modeled by

$$R = 6e^{12.77x},$$

where  $x$  is the blood alcohol concentration and  $R$ , given as a percent, is the risk of having a car accident. What blood alcohol concentration corresponds to a 17% risk of a car accident? How is this shown on the graph of  $R$  in Figure 4.20?

 **Check Point 9** Use the formula in Example 9 to answer this question: What blood alcohol concentration corresponds to a 7% risk of a car accident? (In many states, drivers under the age of 21 can lose their licenses for driving at this level.) 0.01

$$\begin{aligned} \frac{17}{6} &= \frac{6e^{12.77x}}{6} \\ \left(\frac{17}{6}\right) &= e^{12.77x} \\ \ln\left(\frac{17}{6}\right) &= \ln e^{12.77x} \\ \frac{\ln\left(\frac{17}{6}\right)}{12.77} &= \frac{12.77x \cdot \ln e}{12.77} \\ &= x \end{aligned}$$

Suppose that you inherit \$30,000 at age 20. Is it possible to invest \$25,000 and have over half a million dollars for early retirement? Our next example illustrates

**EXAMPLE 10** Revisiting the Formula for Compound Interest

The formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

describes the accumulated value,  $A$ , of a sum of money,  $P$ , the principal, after  $t$  years at annual percentage rate  $r$  (in decimal form) compounded  $n$  times a year. How long will it take \$25,000 to grow to \$500,000 at 9% annual interest compounded monthly?

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t} \\ \frac{500,000}{25,000} &= \frac{25,000\left(1 + \frac{.09}{12}\right)^{12 \cdot t}}{25,000} \\ 20 &= (1.0075)^{12t} \\ \log 20 &= \log 1.0075^{12t} & \ln 20 &= \ln (1.0075)^{12t} \\ \log 20 &= \frac{12t \cdot \log 1.0075}{12 \cdot \log 1.0075} \end{aligned}$$

## Lesson 4-5

**Check Point 2** Strontium-90 is a waste product from nuclear reactions. As a consequence of fallout from atmospheric nuclear tests, we all have a measurable amount of strontium-90 in our bones.

a. The half-life of strontium-90 is 28 years, meaning that after 28 years, the amount of the substance will have decayed to half the original amount. Find the exponential decay model for strontium-90.  $A = A_0 e^{kt}$

b. Suppose that a nuclear accident occurs and releases 60 grams of strontium-90 into the atmosphere. How long will it take for strontium-90 to decay to a level of 10 grams? *about 72 years*

*28 years*

$$A = A_0 e^{kt}$$

a)  $A = A_0 \cdot e^{-0.025t}$

b) 60 = grams at how long  $t$   
10 grams

$$\frac{10}{60} = \frac{60 \cdot e^{-0.025t}}{60}$$

$$\ln \frac{1}{6} = \ln e^{-0.025t}$$

$$\ln \frac{1}{6} = -0.025t$$

$$\frac{\ln \frac{1}{6}}{-0.025} = \frac{-0.025t}{-0.025}$$

$$\ln \frac{1}{6} = -0.025t$$

$$\frac{\ln \frac{1}{6}}{-0.025} = t$$

$$t \approx 72 \text{ years}$$

*In 72 years there would be 10 grams of Strontium-90*

*28 years*

$$\frac{50}{100} = \frac{100 e^{28k}}{100}$$

$$\frac{1}{2} = e^{28k}$$

$$\ln \frac{1}{2} = \ln e^{28k}$$

$$\ln \frac{1}{2} = 28k \cdot \ln e$$

$$\frac{\ln \frac{1}{2}}{28} = \frac{28k}{28}$$

$$-0.025 = k$$

## Logistic Growth Models

From population growth to the spread of an epidemic, nothing on Earth can grow exponentially indefinitely. Growth is always limited. This is shown in Figure 4.23 by the horizontal asymptote. The logistic growth model is a function used to model situations of this type.

## Logistic Growth Model

The mathematical model for limited logistic growth is given by

$$f(t) = \frac{c}{1 + ae^{-bt}} \quad \text{or} \quad A = \frac{c}{1 + ae^{-bt}}$$

where  $a$ ,  $b$ , and  $c$  are constants, with  $c > 0$  and  $b > 0$ .

## EXAMPLE 3 Modeling the Spread of the Flu

The function

$$f(t) = \frac{30,000}{1 + 20e^{-1.5t}}$$

describes the number of people,  $f(t)$ , who have become ill with influenza  $t$  weeks after its initial outbreak in a town with 30,000 inhabitants.

- How many people became ill with the flu when the epidemic began?
- How many people were ill by the end of the fourth week?
- What is the limiting size of  $f(t)$ , the population that becomes ill?

a)  $t=0$   $f(0) = \frac{30,000}{1 + 20e^{-1.5(0)}} = \frac{30,000}{21} = 1429 \text{ People}$

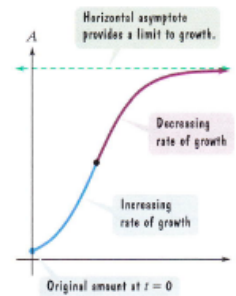
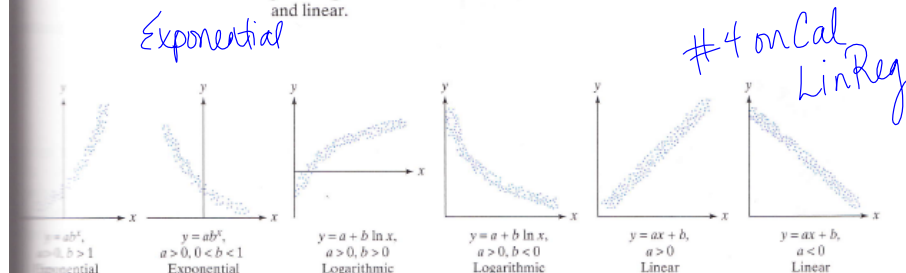


FIGURE 4.23 The logistic growth curve has a horizontal asymptote that identifies the limit of the growth of  $A$  over time.

Choose an appropriate model for data.

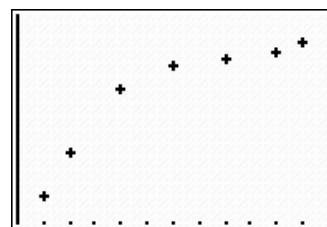
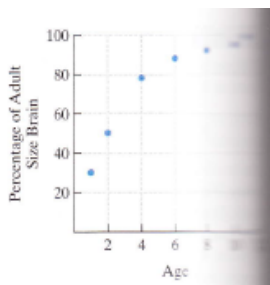
## Modeling Data

Throughout this chapter, we have been working with models that were given. However, we can create functions that model data by observing patterns in scatter plots. Figure 4.24 shows scatter plots for data that are exponential, logarithmic, and linear.



**EXAMPLE 4** Choosing a Model for Data

The data in **Table 4.6** indicate that between the ages of 1 and 11, the human brain does not grow linearly, or steadily. A scatter plot for the data is shown in **Figure 4.25**. What type of function would be a good choice for modeling the data?



```
LnReg
y=a+blnx
a=31.95404756
b=28.94733911
r²=.9806647799
r=.9902852013
```

$$y = 31.95 + 28.95 \ln x$$

**500 Chapter 4** Exponential and L**Table 4.6** Growth of the Human Brain

| Age | Percentage of Adult Size Brain |
|-----|--------------------------------|
| 1   | 30%                            |
| 2   | 50%                            |
| 4   | 78%                            |
| 6   | 88%                            |
| 8   | 92%                            |
| 10  | 95%                            |
| 11  | 99%                            |

Source: Gerrig and Zimbardo, *Psychology and Life*, 18th Edition, Allyn and Bacon, 2008.

**Check Point 4** **Table 4.7** shows the populations of various cities, in thousands, and the average walking speed, in feet per second, of a person living in the city. Create a scatter plot for the data. Based on the scatter plot, what type of function would be a good choice for modeling the data? *logarithmic function\**

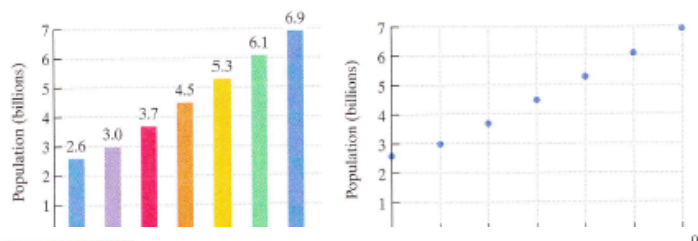
**Table 4.7** Population and Walking Speed

| Population (thousands) | Walking Speed (feet per second) |
|------------------------|---------------------------------|
| 5.5                    | 0.6                             |
| 14                     | 1.0                             |
| 71                     | 1.6                             |
| 138                    | 1.9                             |
| 342                    | 2.2                             |

Source: Mark H. Bornstein and Helen G. Bornstein, "The Pace of Life," *Nature*, 259, Feb. 19, 1976, pp. 557–559.

**EXAMPLE 5** Choosing a Model for Data

**Figure 4.27(a)** shows world population, in billions, for seven selected years from 1950 through 2010. A scatter plot is shown in **Figure 4.27(b)**. Suggest two types of functions that would be good choices for modeling the data.

**World Population, 1950–2010****EXAMPLE 6** Comparing Linear and Exponential Models

The data for world population are shown in **Table 4.10**. Using a graphing utility's linear regression feature and exponential regression feature, we enter the data and obtain the models shown in **Figure 4.28**.

**Table 4.10**

| x, Number of Years after 1949 | y, World Population (billions) |
|-------------------------------|--------------------------------|
| 1 (1950)                      | 2.6                            |
| 11 (1960)                     | 3.0                            |
| 21 (1970)                     | 3.7                            |
| 31 (1980)                     | 4.5                            |
| 41 (1990)                     | 5.3                            |
| 51 (2000)                     | 6.1                            |
| 61 (2010)                     | 6.9                            |

```
LinReg
y=ax+b
a=.0739285714
b=2.293928571
r²=.9931624328
r=.9965753523

ExpReg
y=a*b^x
a=2.576975283
b=1.016951926
r²=.9934466215
r=.9967179248
```

**FIGURE 4.28** A linear model and an exponential model for the data in **Table 4.10**

Because  $r$ , the correlation coefficient, is close to 1 in each screen in **Figure 4.28**, the models fit the data very well.

- Use **Figure 4.28** to express each model in function notation, with numbers rounded to three decimal places.
- How well do the functions model world population in 2000?
- By one projection, world population is expected to reach 8 billion in the year 2026. Which function serves as a better model for this prediction?

\*See Answers to Selected Exercises.

## Attachments

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