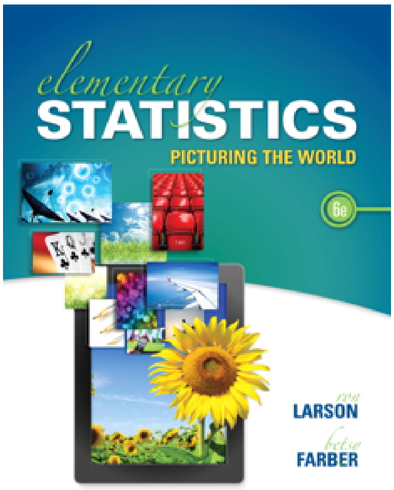
Elementary Statistics: Picturing The World
Sixth Edition



Chapter 3
Probability

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3-1 Basic concepts of Probability and counting

DEFINITION

A **probability experiment** is an action, or trial, through which specific results (counts, measurements, or responses) are obtained. The result of a single trial in a probability experiment is an **outcome.** The set of all possible outcomes of a probability experiment is the **sample space.** An **event** is a subset of the sample space. It may consist of one or more outcomes.

Example 1 Identify the sample sample space of a probability experiment
A probability experiment consists of tossing a coin and then rolling a six-sided die. Determine the number of outcomes and identify the sample space.

solution: TREE DIAGRAM

Example 2 Determine the number of outcomes in each event. Then decide whether each event is simple or not. EXPLAIN

1. For quality control, you randomly select a machine part from a batch that has been manufactured that day. Even A is selecting specific defective machine part.

2. You roll a six-sided die. Event B is rolling at least a 4

solution: Event A has only one outcome: Choosing a specific defective machine part so the event is a SIMPLE event

solution: Event B has three outcomes: Rolling a 4, 5 or 6. Because the event has more than one outcome NOT SIMPLE

THE FUNDAMENTAL COUNTING PRINCIPLE

If one event can occur in m ways and a second event can occur in n ways, the number of ways the two events can occur in sequence is $m \cdot n$. This rule can be extended for any number of events occurring in sequence.

Example 3 Using the fundamental counting principle
You are purchasing a new car. The possible manufacturers, car sizes, and colors are listed
Manufacturers: Ford, FM, Honda Car size: Compact, Midsize Color: White, Red, Black, Green
How many different ways can you select one manufacturer, one car size, and one color? Use a tree diagram

solution:

Example 4: Using the fundamental counting principle

The access code for a car's security system consists of four digits. Each digit can be 0 through 9

___ ___ ___ ___ How many access codes are possible if

1. Each digit can be used only once and not repeated

2. Each digit can be repeated

3. Each digit can be repeated but the first one can't be 0 or 1

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DEFINITION

Classical (or **theoretical**) **probability** is used when each outcome in a sampl space is equally likely to occur. The classical probability for an event E i given by

$$P(E) = \frac{\text{Number of outcomes in event } E}{\text{Total number of outcomes in sample space}}.$$

Example 5 Finding Classic probabilities
You roll a die. Find the probability of each event
1. Event A: Rolling a 3
2. Event B: Rolling a 7
3. Event C: Rolling a number less than 5

solution:

DEFINITION

Empirical (or **statistical**) **probability** is based on observations obtained from probability experiments. The empirical probability of an event E is the relative frequency of event E.

$$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}} = \frac{f}{n}.$$

When an experiment is repeated many times, regular patterns are formed. These patterns make it possible to find empirical probability. Empirical probability can be used even if each outcome of an event is not equally likely to occur.

Example 6 Finding Empirical Probabilities

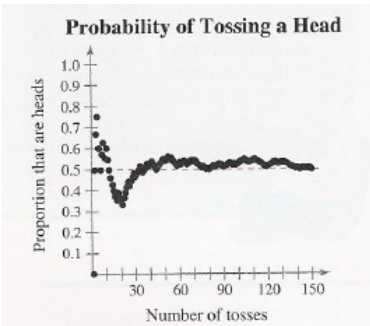
A company is conducting an online survey of randomly selected individu to determine if traffic congestion is a problem in their community. So far, 3 people have responded to the survey. The frequency distribution shows t results. What is the probability that the next person that responds to the surv says that traffic congestion is a serious problem in their community?

Response	Number of times, f
It is a serious problem.	123
It is a moderate problem.	115
It is not a problem.	82
	$\Sigma f = 320$

solution: The event is a response of "it is a serious problem" The frequency of this event is 123. The total is 320 so...
P(Serious Problem)=

LAW OF LARGE NUMBERS

As an experiment is repeated over and over, the empirical probability of an event approaches the theoretical (actual) probability of the event.



Example 7 Using Frequency distributions to find probability

Employee ages	Frequency, f
15 to 24	54
25 to 34	366
35 to 44	233
45 to 54	180
55 to 64	125
65 and over	42
	$\Sigma f = 1000$

You survey a sample of 1000 employees at a company and record the age of each. The results are shown at the left in the frequency distribution. If you randomly select another employee, what is the probability that the employee will be between 25 and 34 years old?

solution: The event is selecting an employee who is between 25 and 34 years old. In your survey, the frequency of this event is 366. Because the total of the frequencies is 1000, the probability of selecting an employee between 25-34 is
P(age 25-34)=

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The third type of probability is **subjective probability.** Subjective probabilities result from intuition, educated guesses, and estimates. For instance, given a patient's health and extent of injuries, a doctor may feel that the patient has a 90% chance of a full recovery. Or a business analyst may predict that the chance of the employees of a certain company going on strike is 0.25.

Example 8 Classifying types of probabilities

Classify each statement as an example of Classical probability, Empirical Probability or Subjective probability EXPLAIN EACH!!!

1. The prob that you will be married by the age 30 is 0.5
2. The prob that a voter chosen tat random will vote Republican is 0.45
3. The prob of winning a 1000-ticket raffle with one ticket is 1/1000

solution: The prob is most likely based on an educated guess. It is an example of Subjective prob

solution: This statement is most likely based on a survey of a sample of voters, so it is an example of empirical

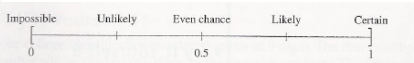
solution: Because you know the number of outcomes and each is equally likely, this is an example of classical

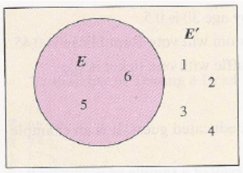
A probability cannot be negative or greater than 1. So, the probability of an event E is between 0 and 1, inclusive, as stated in the following rule.

RANGE OF PROBABILITIES RULE

The probability of an event E is between 0 and 1, inclusive. That is,

$0 \le P(E) \le 1.$





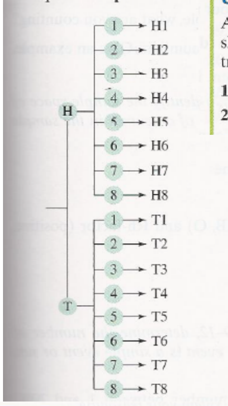
The area of the rectangle represents the total probability of the sample space (1 = 100%). The area of the circle represents the probability of event E, and the area outside the circle represents the probability of the complement of event E.

DEFINITION

The **complement of event** E is the set of all outcomes in a sample space that are not included in event E. The complement of event E is denoted by E' and is read as "E prime."

Example 9 Use example 7 data. Find the probability of randomly choosing an employee who is NOT between 25-34 years old

Tree Diagram for Coin and Spinner Experiment

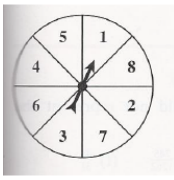


EXAMPLE 10

Using a Tree Diagram

A probability experiment consists of tossing a coin and spinning the spinner shown to the left. The spinner is equally likely to land on each number. Use a tree diagram to find the probability of each event.

1. Event A: tossing a tail and spinning an odd number
2. Event B: tossing a head or spinning a number greater than 3



EXAMPLE 11

Using the Fundamental Counting Principle

Your college identification number consists of 8 digits. Each digit can be 0 through 9 and each digit can be repeated. What is the probability of getting your college identification number when randomly generating eight digits?

Solution Because each digit can be repeated, there are 10 choices for each of the 8 digits. So, using the Fundamental Counting Principle, there are $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^8 = 100{,}000{,}000$ possible identification numbers.

But only one of those numbers corresponds to your college identification number. So, the probability of randomly generating 8 digits and getting your college identification number is 1/100,000,000.

▸ Try It Yourself 11

Your college identification number consists of 9 digits. The first two digits (number will be the last two digits of the year you graduate. The other digits 0 through 9 and each digit can be repeated. What is the probability of gettin college identification number when randomly generating the other seven

a. Find the *total number* of possible identification numbers. Assume th are scheduled to graduate in 2012.

b. Find the *probability* of randomly generating your identification num

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Conditional probability and the multiplication Rule pg. 149

Conditional Probability: probability that two events occur in sequence

DEFINITION

A **conditional probability** is the probability of an event occurring, given that another event has already occurred. The conditional probability of event B occurring, given that event A has occurred, is denoted by $P(B|A)$ and is read as "probability of B, given A."

Example 1 Finding conditional Probabilities

1. Two cards are selected in sequence from a standard deck. Find the probablity that the second card is a queen, given that the first card is a King. (Assume that the king is NOT replaced)

Solution: 52 cards in a deck $P(B|A)=\frac{4}{51}=0.078$

2. The table shows the results of a study in which researchers examined a Child's IQ and th presence of a certain gene in the child. Find the probability that a child has a high IQ, given that the child has the gene.

Solution $P(B|A)=\frac{33}{72}=0.458$

Sample Space

	Gene present
High IQ	33
Normal IQ	39
Total	72

	Gene present	Gene not present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

Independent and Dependent Events: In some experiments one event doesn't affect the probability of the other. Rolling a die and then flipping a coin. THEY ARE INDEPENDENT. The question of the independence of two or more events is important. You can use conditional probs to determine whether events are independent or not

DEFINITION

Two events are **independent** if the occurrence of one of the events does not affect the probability of the occurrence of the other event. Two events A and B are independent if

$$P(B|A)=P(B) \quad \text{or if} \quad P(A|B)=P(A).$$

Events that are not independent are **dependent.**

to determine if A and B are INDEPENDENT, first calculate P(B), the probablity of event B. Then Calculate P(B|A). The prob of B, given A. If the values are equal, then the the events are independent.

Example 2 Decide whether the events are independent or dependent

1. Selecting a king from a standard deck (A), not replacing it and then selecting a queen from the deck (B) $P(B|A)=\frac{4}{51}$ $P(B)=\frac{4}{52}$

Solution:

The occurrence of A changes the prob B, so the events are dependent

2. Tossing a coin and getting a head (A), and then rolling a six-sided die and rolling a 6 (B)

Solution: $P(B|A)=\frac{1}{6}$ $P(B)=\frac{1}{6}$

The occurrence of A doesn't change the prob of the occurrence of B so the events are independent

3. Driving over 85 miles an hour (A), and then getting in a car accident (B)

Solution: if you are driving over 85 mph, the chances of getting in a car accident are greatly increased, so these events are dependent

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Mulitplication Rule pg. 151

THE MULTIPLICATION RULE FOR THE PROBABILITY OF A AND B

The probability that two events A and B will occur in sequence is

$P(A \text{ and } B) = P(A) \cdot P(B|A)$.

If events A and B are independent, then the rule can be simplified to $P(A \text{ and } B) = P(A) \cdot P(B)$. This simplified rule can be extended for any number of independent events.

Example 3: Using the multiplication rule to find probablities

1. Two cards are selected, without replacement the first card, from a standard decfk. Find the prob of selecting a king and then a queen

$$P(K \,\&\, Q) = P(K) \cdot P(Q \mid K)$$

2. A coin is tossed and a die is rolled. Find the prob of getting a head and then rolling a 6

$$P(H \,\&\, 6) = P(H) \cdot P(6)$$

3. Prob that a salmon swims successfully through a dam is 0.85. Find the prob that two salmon successfully swim through the dam

EXAMPLE 4

Using the Multiplication Rule to Find Probabilities

1. A coin is tossed and a die is rolled. Find the probability of getting a tail and then rolling a 2.
2. The probability that a particular knee surgery is successful is 0.85. Find the probability that three knee surgeries are successful.
3. Find the probability that none of the three knee surgeries is successful.
4. Find the probability that at least one of the three knee surgeries is successful.

Solution

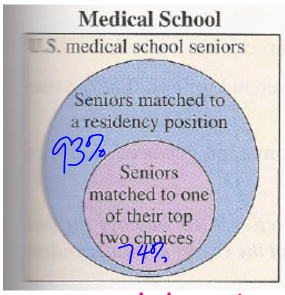
Statistics Lesson 3-2 continued

Date:____

EXAMPLE 5

Using the Multiplication Rule to Find Probabilities

More than 15,000 U.S. medical school seniors applied to residency programs in 2007. Of those, 93% were matched to a residency position. Seventy-four percent of the seniors matched to a residency position were matched to one of their top two choices. Medical students electronically rank the residency programs in their order of preference and program directors across the United States do the same. The term "match" refers to the process where a student's preference list and a program director's preference list overlap, resulting in the placement of the student for a residency position. *(Source: National Resident Matching Program)*



1. Find the probability that a randomly selected senior was matched A to a residency position *and* it was one of the senior's top two choices. B
2. Find the probability that a randomly selected senior that was matched to a residency position did *not* get matched with one of the senior's top two choices.
3. Would it be unusual for a randomly selected senior to result in a senior that was matched to a residency position and it was one of the senior's top two choices? ← compliment

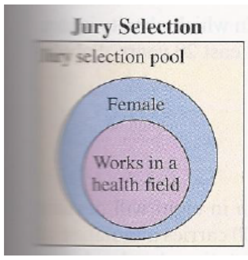
1) A = {Matched to res pos}
B = {match to top 2}
P(A and B) = P(A) = .93
P(B|A) = .74
P(A & B) = .93 · .74 = .688
** Prob a random senior was Matched in top 2 choices .688

2) $P(B'|A) = 1 - P(B|A) = 1 - .74 = .26$
the prob you got your residency but Not in top 2 is .26

3) $1 - .26 = .74$

▸ Try It Yourself 5

In a jury selection pool, 65% of the people are female. Of these 65%, one out of four works in a health field.



1. Find the probability that a randomly selected person from the jury pool is female and works in a health field.
2. Find the probability that a randomly selected person from the jury pool is female and does not work in a health field.

a. Determine *events A* and *B*.
b. Use the *Multiplication Rule* to write a formula to find the probability. If necessary, use the *Complement Rule*.
c. *Calculate* the probability. *Answer: Page A38*

➡ Assign: pg.152
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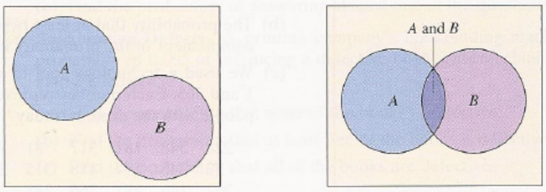
The Addition Rule pg. 160

DEFINITION

Two events A and B are **mutually exclusive** if A and B cannot occur at the same time.

The Venn diagrams show the relationship between events that are mutually exclusive and events that are not mutually exclusive.



A and *B* are mutually exclusive. *A* and *B* are not mutually exclusive.

Example 1: Mutually exclusive events

Decide if the events are mutually exclusive. EXPLAIN your reasoning

1. Event A : Roll a 3 on a die
   Event B: Roll a 4 on a die

Solution: The first event has one outcome a 3, Event B also has one outcome a 4, These outcomes cannot occur at the same time, so The events are mutually exclusive

2. Event A: Randomly select a male
   Event B: Randomly select a nursing major

Solution: Because the students can be a male nursing major, the events are NOT mutually exclusive

3. Event A: Randomly select a blood donor with type O blood
   Event B: Randomly select a female blook donor

Solution:Because the donor can be a female with type O blood, the events are NOT mutually exclusive

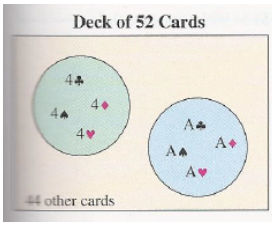
THE ADDITION RULE FOR THE PROBABILITY OF *A* OR *B*

The probability that events A or B will occur, $P(A \text{ or } B)$, is given by

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

If events A and B are mutually exclusive, then the rule can be simplified to $P(A \text{ or } B) = P(A) + P(B)$. This simplified rule can be extended to any number of mutually exclusive events.

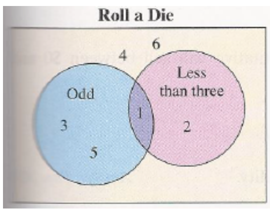
In words, to find the probability one event or the other will occur, add the individual probabilities of each event and subtract the probability they both occur.



Example 2: Using the addition Rule to find probabilities

1.You select a card from a standard deck. Find the probability that the card is a 4 or an ace

Solution:



2. You roll a die. Find the probability of rolling a number less tha three or rolling an odd number

Solution:

EXAMPLE 3

Finding Probabilities of Mutually Exclusive Events

The frequency distribution shows the volume of sales (in dollars) and the number of months a sales representative reached each sales level during the past three years. If this sales pattern continues, what is the probability that the sales representative will sell between \$75,000 and \$124,999 next month?

Sales volume (\$)	Months
0–24,999	3
25,000–49,999	5
50,000–74,999	6
75,000–99,999	7
100,000–124,999	9
125,000–149,999	2
150,000–174,999	3
175,000–199,999	1

Solution:

A = monthly sales between \$75,000 and \$99,999

B = monthly sales between \$100,000 and \$124,999

Statistics Lesson 3-3 Continued

Date:_____

EXAMPLE 4

Using the Addition Rule to Find Probabilities

A blood bank catalogs the types of blood, including positive or negative Rh-factor, given by donors during the last five days. The number of donors who gave each blood type is shown in the table. A donor is selected at random.

1. Find the probability that the donor has type O or type A blood.
2. Find the probability that the donor has type B blood or is Rh-negative.

		Blood Type				
		O	A	B	AB	Total
RH-factor	Positive	156	139	37	12	344
	Negative	28	25	8	4	65
	Total	184	164	45	16	409

Solution:

1. Because a donor cannot have type O blood and type A blood, these events are mutually exclusive. So, on the basis of the Addition Rule, the probability that a randomly chosen donor has type O or type A blood is

$$P(\text{type O or type A}) = P(\text{type O}) + P(\text{type A})$$

2. Because a donor can have type B blood and be Rh-negative, these events are not mutually exclusive. So, on the basis of the Addition Rule, the probability that a randomly chosen donor has type B blood or is Rh-negative is

$$P(\text{type B or Rh-neg}) = P(\text{type B}) + P(\text{Rh-neg}) - P(\text{type B and Rh-neg})$$

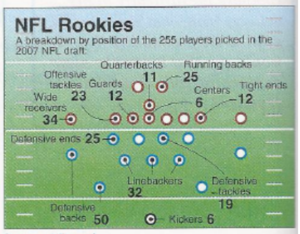
A Summary of Probability

Type of Probability and Probability Rules	In Words	In Symbols
Classical Probability	The number of outcomes in the sample space is known and each outcome is equally likely to occur.	$P(E) = \dfrac{\text{Number of outcomes in event } E}{\text{Number of outcomes in sample space}}$
Empirical Probability	The frequency of outcomes in the sample space is estimated from experimentation.	$P(E) = \dfrac{\text{Frequency of event } E}{\text{Total frequency}} = \dfrac{f}{n}$
Range of Probabilities Rule	The probability of an event is between 0 and 1, inclusive.	$0 \le P(E) \le 1$
Complementary Events	The complement of event E is the set of all outcomes in a sample space that are not included in E, denoted by E'.	$P(E') = 1 - P(E)$
Multiplication Rule	The Multiplication Rule is used to find the probability of two events occurring in a sequence.	$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ *Independent events*
Addition Rule	The Addition Rule is used to find the probability of at least one of two events occurring.	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = P(A) + P(B)$ *Mutually exclusive events*

EXAMPLE 5

Combining Rules to Find Probabilities

Use the graph at the right to find the probability that a randomly selected draft pick is not a running back or a wide receiver.



(Source: NFL.com)

Solution Define events A and B.

A: Draft pick is a running back.
B: Draft pick is a wide receiver.

These events are mutually exclusive, so the probability that the draft pick is a running back or wide receiver is

Try It Yourself 5

Find the probability that a randomly selected draft pick is not a linebacker or a quarterback.

a. Find the *probability* that the draft pick is a linebacker or a quarterback.
b. Find the *complement* of the event.

Answer: Page A38

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Statistics Lesson 3-4 Date:________

Permutations, combinations and applications

DEFINITION

A **permutation** is an ordered arrangement of objects. The number of different permutations of n distinct objects is $n!$.

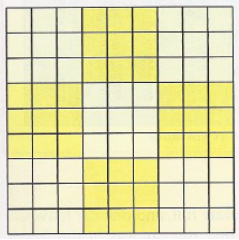
The expression $n!$ is read as n **factorial** and is defined as follows.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots 3 \cdot 2 \cdot 1$$

As a special case, $0! = 1$. Here are several other values of $n!$.

$$1! = 1,\ 2! = 2 \cdot 1 = 2,\ 3! = 3 \cdot 2 \cdot 1 = 6,\ 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Sudoku Number Puzzle



Example 1: The objective of a 9X9 sudoku number puzzle is to fill in the grid so that each row, each column, and each 3X3 grid contains the digits 1 to 9. How many different ways can the first row of a blank 9X9 grid be filled?

Solution:

PERMUTATIONS OF n OBJECTS TAKEN r AT A TIME

The number of permutations of n distinct objects taken r at a time is

$${}_nP_r = \frac{n!}{(n-r)!}, \text{ where } r \le n.$$

Example 2: Find the number of ways of forming three-digit codes in which no digit is repeated

Solution: ${}_nP_r = {}_{10}P_3 = \dfrac{10!}{(10-3)!} =$

Example 3: Permutation of n objects taken r at a time

43 race cars started the 2007 Daytona 500. How many ways can the car finish 1st, 2nd, 3rd

Solution:

You may want to order a group of n objects in which some of the objects are the same. For instance, consider a group of letters consisting of four As, two Bs, and one C. How many ways can you order such a group? Using the previous formula, you might conclude that there are ${}_7P_7 = 7!$ possible orders. However, because some of the objects are the same, not all of the permutations are *distinguishable*. How many distinguishable permutations are possible? The answer can be found using the following formula.

Study Tip

The letters *AAAABBC* can be rearranged in 7! orders, but many of these are not distinguishable. The number of distinguishable orders is

$$\frac{7!}{4! \cdot 2! \cdot 1!} = \frac{7 \cdot 6 \cdot 5}{2} = 105.$$

DISTINGUISHABLE PERMUTATIONS

The number of **distinguishable permutations** of n objects, where n_1 are of one type, n_2 are of another type, and so on is

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots n_k!}, \text{ where } n_1 + n_2 + n_3 + \cdots + n_k = n.$$

Example 4 distinguishable permutations

a building contractor is planning to develop a subdivision. The subdivision is to consist of 6 one-story houses, 4 two-story houses, and 2 split level houses. IN how many distinguishable ways can the house be arranged?

Solution: There are to be 12 houses in the subdivision, 6 of which are of one type, 4 of two-story, and 2 of the third type (split level) So

$$\frac{12!}{6! \cdot 4! \cdot 2!} =$$

Statistics Lesson 3-4 continued

Combinations

You want to buy three CDs from a selection of five CDs. There are 10 ways to make your selections.

ABC, ABD, ABE,
ACD, ACE,
ADE,
BCD, BCE,
BDE,
CDE

In each selection, order does not matter (ABC is the same set as BAC). The number of ways to choose r objects from n objects without regard to order is called the number of **combinations of n objects taken r at a time.**

COMBINATION OF n OBJECTS TAKEN r AT A TIME

A combination is a selection of r objects from a group of n objects without regard to order and is denoted by ${}_nC_r$. The number of combinations of r objects selected from a group of n objects is

$${}_nC_r = \frac{n!}{(n-r)!r!}.$$

Example 5 Finding the number combinations

A state's department of transportation plans to develop a new section of interstate highway and receives 16 bids for the project. The state plans to hire four of the bidding companies. How many combinations of four companies can be selected from the 16 companies?

Solution: The state is selecting four companies from a group of 16, so n=16 and r=4/ Because order is NOT important

$${}_nC_r = {}_{16}C_4 = \frac{16!}{(16-4)!4!}$$

Principle	Description	Formula
Fundamental Counting Principle	If one event can occur in m ways and a second event can occur in n ways, the number of ways the two events can occur in sequence is $m \cdot n$.	$m \cdot n$
Permutation	The number of different ordered arrangements of n distinct objects	$n!$
	The number of permutations of n distinct objects taken r at a time, where $r \leq n$	${}_nP_r = \frac{n!}{(n-r)!}$
	The number of distinguishable permutations of n objects where n_1 are of one type, n_2 are of another type, and so on	$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}$
Combinations	The number of combinations of r objects selected from a group of n objects without regard to order	${}_nC_r = \frac{n!}{(n-r)!r!}$

EXAMPLE 6

Finding Probabilities

A student advisory board consists of 17 members. Three members serve as the board's chair, secretary, and webmaster. Each member is equally likely to serve any of the positions. What is the probability of selecting at random the three members that hold each position?

EXAMPLE 7

Finding Probabilities

You have 11 letters consisting of one M, four Is, four Ss, and two Ps. If the letters are randomly arranged in order, what is the probability that the arrangement spells the word *Mississippi*?

EXAMPLE 8

Finding Probabilities

Find the probability of being dealt five diamonds from a standard deck of playing cards. (In poker, this is a diamond flush.)

Solution The possible number of ways of choosing 5 diamonds out of 13 is ${}_{13}C_5$. The number of possible five-card hands is ${}_{52}C_5$. So, the probability of being dealt 5 diamonds is

EXAMPLE 9

Finding Probabilities

A food manufacturer is analyzing a sample of 400 corn kernels for the presence of a toxin. In this sample, three kernels have dangerously high levels of the toxin. If four kernels are randomly selected from the sample, what is the probability that exactly one kernel contains a dangerously high level of the toxin?

Solution

The possible number of ways of choosing one toxic kernel out of three toxic kernels is ${}_3C_1$. The possible number of ways of choosing 3 nontoxic kernels from 397 nontoxic kernels is ${}_{397}C_3$. So, using the Fundamental Counting Principle, the number of ways of choosing one toxic kernel and three nontoxic kernels is

Assign: pg.174 3-6,12, 16,18,20,25,29,34,41,42, 49,59-62

Attachments

- Lesson 3-1 Book.pdf
- Lesson 3-2 Book.pdf
- Lesson 3-3 Book.pdf
- Lesson 3-4 Notes.pdf
- Stats Ch 3 Real Stats.pdf
- Poker Probablity